# A Cadence of Change

### The Rise and Fall of the Ionian-ization of Minor

## Part I — The history

### 1. What happened

Between the fourteenth century and the eighteenth, the final cadence of minor-mode Western music was rebuilt in three datable changes, arriving in sequence, separated by generations, in strictly decreasing order of resemblance to Ionian's cadence. This paper names the three changes for the three changes' results: the **Dominant-ization** (the raised seventh, giving the cadential pre-tonic dominant quality; established by the fourteenth century), the **Major-ization** (the major third imposed at the cadential tonic; complete by about 1565), and the **Fortification** (the augmented sixth added at the cadential pre-pre-tonic; established by about 1740). Each change, once established, remained available thereafter, so that by the mid-eighteenth century the fully rebuilt form — augmented sixth, dominant seventh, major tonic — existed as a cadence-type, available and describable though never frequent as a complete final gesture (§4, §6), containing no chord that Dorian, Aeolian, or Phrygian supplies.

The three changes then came apart, and the three changes came apart along three different curves. The Major-ization collapsed between roughly 1700 and 1850 — from totals to about 14 percent — and the collapse began where church style did not reach: in the keyboard and chamber repertoire of 1700–1724, the Picardy third already holds only 16.7 percent of closed minor endings while the contemporaneous chorale repertoire holds 94.2, a genre split measured inside a single generation. The two endpoints measured since — Chopin's 14.3 percent and modern popular song's 13.2 — are joined by a measured floor: every quarter-century bin from 1850 to 1949 sits between 0 and 18 percent. The Dominant-ization never collapsed: the raised seventh stands today within three points of the raised seventh's notated rate around 1600 (Tasso corpus, 1590–1609). The Fortification was still rising while the Major-ization's collapse ran, the two series carried by the same keyboard corpora from Scarlatti forward. And the seven-mode field that the Ionian-ization had pruned — Lydian reclassified out of existence, Locrian excluded, Dorian and Phrygian marginalized — has not refilled in any corpus measured here, the modern endpoint included: Ionian and Aeolian together hold nine-tenths of modern popular song.

Alongside the composed record runs a second record: the teaching literature. At no measured transition did the teaching literature move ahead of the leading edge of composition. The written record trails the composed record, usually by a generation and at the nearest writing from inside a change's window (§7) — naming each change only after composers had established the change, and carrying one dead convention in classrooms for two centuries after composers abandoned the convention — a teaching lineage documented from Bach's own classroom to the 1941 Schirmer edition (§7).

Part I documents all of the above — rise, fall, and teaching record — in the language of observation: *appears*, *completes*, *rises*, *falls*, *is absent*. Part I contains no causal claims. Why the sequence rose, and why the sequence fell apart in exactly three ways, is a different order of claim, argued in Part II as a conjecture ranked against rival explanations. Part III draws out what follows if the account is right — including the finding that two of the objects modern pedagogy teaches as *scales*, harmonic minor and melodic minor, do not operate as scales in the repertories sampled here — and names the research that remains, most of which no amount of computation can do.

The evidence base is roughly 32,000 encoded works and movements across eleven corpus groups — the trecento repertory (14th c.), the Josquin Research Project (c. 1420–1550), CRIM (Citations: The Renaissance Imitation Mass; c. 1500–1610, per-piece dates), the Palestrina corpus (c. 1554–94, modes

declared), the Tasso in Music Project (1571–1649, per-work publication dates), the Corelli and Vivaldi instrumental corpora (published opus sets, 1681–1714), the Bach chorales, the Scarlatti, Haydn, Mozart, and Beethoven keyboard sonatas, the Chopin preludes and mazurkas, the DCML Distant Listening Corpus (expert-annotated movements, c. 1600–1949, per-piece composition dates), and the Hooktheory corpus of modern popular song (26,175 entries, modes declared) — together with the theoretical and pedagogical sources of each period, cited at page level wherever a page could be verified (Appendix A). Appendix B specifies the corpora; Appendix C the operational definitions and statistical conventions; Appendix D the full result tables; Appendix E the editorial-provenance accounting; Appendix F what is known about detector and classifier reliability; Appendix G the reproduction record.

## 2. Terms

The following terms are used throughout, one term per concept.

**Final cadence** — the closing progression of a piece: the last chord and the chords immediately preceding the last chord.

**Cadential tonic** — the last chord of the final cadence.

**Cadential pre-tonic** — the chord immediately before the cadential tonic. In common-practice minor the cadential pre-tonic is the dominant seventh.

**Cadential pre-pre-tonic** — the chord immediately before the cadential pre-tonic. In common-practice minor the cadential pre-pre-tonic is where the augmented sixth appears.

These three positional terms are coined here. The literature supplies partial equivalents — *ultima* and *penultima* in Renaissance cadence theory, *dominant* and *predominant* in functional harmony — but the two vocabularies belong to different systems and neither vocabulary extends cleanly to the third position.

**Modal** — supplied by the mode in which the piece is written.

**Extra-modal** — not supplied by the mode in which the piece is written.

**Extra-diatonic** — supplied by no diatonic mode at all. An extra-diatonic feature is extra-modal in every mode; the reverse does not hold.

**Minor-destination modes** — Dorian, Aeolian and Phrygian: the three modes with a minor third and a perfect fifth above the final. Common-practice minor descends from Dorian, Aeolian and Phrygian.

**Ionian-izing** — a graded property of an alteration: the degree to which the feature the alteration installs identifies Ionian. An alteration is maximally Ionian-izing when Ionian alone natively possesses the installed feature, moderately so when the feature is shared with other modes, and not Ionian-izing at all when no mode — Ionian included — possesses the feature.

**Law** — a convention observed without exceptions. A law's compliance rate sits at or near one hundred percent, which distinguishes a law from a strong preference: a preference produces seventy percent, and a law produces totals — 128 of 128, 574 of 580.

**Choice** — a convention applied where a stated condition holds and withheld where the condition fails. A choice's rate floats with the frequency of the condition, while the conditional rate — apply-given-condition — stays stable.

Two further terms are standard and used in their standard sense: **Picardy third**, the major third at the cadential tonic of a minor-destination piece; and **augmented sixth**, the chord built on the flat sixth degree with a raised fourth above the flat sixth degree.

Each of the three modifications documented below is called a **change**:

**Dominant-ization** — the first change: the raising of the seventh degree at the cadential pre-tonic. Raising the seventh beside a flat sixth creates an augmented second, singers removed the augmented second by raising the sixth as well, and the raised sixth is therefore part of the Dominant-ization (§4). The Dominant-ization's result is a cadential pre-tonic of dominant quality — in mature chordal form, dominant-seventh quality.

**Major-ization** — the second change: the imposition of a major third at the cadential tonic — the Picardy third made obligatory.

**Fortification** — the third change: the addition of the augmented sixth at the cadential pre-pre-tonic, preparing the Dominant-ized cadential pre-tonic.

**De-Ionian-ization** — the coming-apart documented in §6.

*The milestone vocabulary*. Four words carry dates in this paper, and each is defined against its change's own unit, because the three changes are measured in unlike units — notes at a scale degree for the Dominant-ization, piece endings for the Major-ization, verticalities for the Fortification — and no single percentage threshold can order quantities that are not commensurable. **Arrived** — the feature is attested above the detection-noise level in at least one dated corpus, the noise level being the rate produced by known false-positive sources in that corpus, which for the augmented sixth is the editorial-accidental and cross-relation rate adjudicated in §4. **Established** — the feature is cadentially functional rather than incidental: it occurs at cadences, resolves as the cadence requires, and is not confined to a single composer or genre in the corpus that first shows it. For the augmented sixth this criterion is met at phrase level, where the measurement belongs (§4). **Effectively obligatory**, equivalently a **law** — compliance at or near totals. **Leading edge** — the earliest-dated corpus in which the feature meets the relevant criterion, named as a corpus rather than as a repertory at large, since the corpora sampled here are not a representative survey of composition.

*The hyphenated-neologisms rule*. Terms coined in this paper are hyphenated — Ionian-ization, Ionian-izing, Dominant-ization, Major-ization, De-Ionian-ization, extra-modal, extra-diatonic — so that the coinage stays visible on the page. Established words, Fortification among them, are not.

## 3. The object measured

The rebuilt object is the final cadence of a minor-destination piece in the fully armed form the sequence assembled — every element established as practice by about 1740, the assembled form never the majority ending (§4): a cadential pre-pre-tonic driving onto the cadential pre-tonic by contrary semitones; a cadential pre-tonic whose internal tritone resolves onto the cadential tonic's frame, raised seventh rising a semitone to the tonic and fourth degree falling a semitone to the third; and a cadential tonic carrying a major third.

Measured against the minor-destination modes, every functional element of that final cadence is extra-modal. The raised seventh is extra-modal. The Picardy third is extra-modal. The augmented sixth is extra-diatonic — supplied by no diatonic mode whatever, and therefore extra-modal in every mode. The raised sixth that smooths the melodic path to the raised seventh is extra-modal as well — though §4 shows that the raised sixth is not an independent change, arriving instead as a consequence of the Dominant-ization.

The finding is when each element arrived, at which cadential position, in what order, how far apart — and then when, and in what manner, each element departed.

## 4. The rise

### The Dominant-ization — the raised seventh, and the raised seventh's melodic repair. Established practice by the fourteenth century. Position: cadential pre-tonic.

The cadential inflection that raises the seventh degree — *musica ficta* (performer-supplied accidentals), applied under the rubric *causa pulchritudinis* (inflection for beauty) — is attested repeatedly from the fourteenth century onward; the rubric pairing (inflection *causa necessitatis*, to remove a forbidden interval, against inflection *causa pulchritudinis*, to sharpen a cadence) runs through the fifteenth-century sources: the anonymous treatise tradition quoted in Bent (2002, 78–79), the survey in Berger (1987), and the manuscript study of Hughes (1972). Marchetto of Padua (*Lucidarium*, 1317–18) goes further, dividing the whole tone into five dieses and assigning the cadential chromatic inflection four of the five, so that the inflected approach to the goal note is narrower than a natural semitone (Lucidarium 2.4–2.8, 5.6; Herlinger 1985, 141–55, 208–23).

In the corpora the inflection appears as the raised-seventh share of seventh-degree notes in minor-destination pieces: 5.2–8.4% in the fifteenth-century cohorts, rising to 12.3–15.3% by 1560–1649. That series requires an immediate qualification, measured in Appendix E, because the qualification changes what the series means. In the Josquin corpus almost every raised seventh counted is an accidental supplied by a modern editor rather than written by a scribe: removing editorially supplied accidentals collapses the fifteenth-century rate from 5.2–8.4% to 0.0–0.4%. In the Tasso corpus the same removal barely moves the rate, from 12.3–15.3% to 9.3–12.6%. The notated rise is therefore real and far larger than the raw series shows — roughly thirty-fold rather than two-fold — but a substantial part of what rises is the raised seventh passing from unnotated, left to the performer as *musica ficta*, into notation. This paper cannot separate the change in notational practice from the change in musical practice, and does not claim to. The Dominant-ization precedes both other changes by at least a century and a half on the documentary and cadential evidence; the corpus rates are consistent with that precedence but, given the notational confound, do not independently establish it.

**The melodic repair.** Raising the seventh in a mode with a flat sixth creates an augmented second between the two degrees, an interval the repertories all but never traverse: once in 341,005 notes of JRP's Aeolian and Phrygian pieces, never in Palestrina's 153,963, seven times in Tasso's 76,445. The repair is to raise the sixth as well when a line passes beside the raised seventh. Pedagogy treats the raised sixth as a separate development, the "melodic minor" adjustment. The corpora do not: the rule *raise the sixth when a raised seventh is adjacent* operates at 79.7–93.9% in every corpus from 1420 to 1750 when accidentals are counted as printed, and the dependence survives the removal of editorial accidentals — in purely scribal notation the rule still fires at 74.1% in the Josquin corpus against a far-from-the-seventh baseline of 0.5%, and at 56.1% in Tasso against 18.4% (Appendix D, Table D3). The raised sixth tracks the raised seventh under every accounting. What the accounting does change is the rule's measured strength: under the scribal-only policy the conditional rate rises across the Tasso periods, from 26.3% (1560–89) through 61.0% to 66.7% (1610–49), a rise this evidence cannot apportion between notational practice — early under-notation of the sixth — and sung practice (Appendix D, Table D4; Appendix E). The raised sixth is therefore best read not as an independent change but as the Dominant-ization's melodic consequence: dependence the conditional tables support strongly under both policies, with the dating of the dependence's full strength an open question that only newly sourced manuscripts could close.

### The Major-ization — the Picardy third. Under 5% before 1520; complete by c. 1565. Position: cadential tonic.

The cadential tonic falls in three stages, and the middle stage is what makes the third stage significant.

**Stage one: no third at all.** In the fourteenth-century trecento corpus, 99.0% of pieces (102 of 103) end on a cadential tonic containing no third — a bare octave or fifth. The question *major or minor?* cannot yet be asked.

**Stage two: thirds enter, and the entering thirds are minor.** Across the fifteenth century the no-third share of minor-destination endings falls from 93.5% (composers born 1420–39) to 72.4% (born 1440–59). The thirds that enter are the modal thirds: in the 1440–59 birth cohort, 26.3% of all minor-destination endings carry a minor third and 1.2% a major third, the remainder closing without any third.

**Stage three: the third becomes obligatorily major.** Among endings that carry a third: 4.4% major in the 1440–59 birth cohort (7 of 159, JRP); 99.0% in Palestrina (574 of 580, c. 1554–94); 100.0% (128 of 128), 97.4% (38 of 39) and 100.0% (15 of 15) in the Tasso corpus through 1649. From under five per cent to categorical inside roughly one generation, c. 1540–1565. The CRIM corpus, dated per piece and independent of Palestrina and Tasso, corroborates: among minor-destination pieces carrying a third at the close, 0 of 2 in 1500–34, 3 of 6 in 1535–54, 51 of 59 in 1555–74, 17 of 17 in 1575–1609 — with the caveat that CRIM mode inference disagrees with the traditional ambit criterion in 38.1% of pieces, so CRIM corroborates the transition and does not establish it; the Palestrina and Tasso corpora carry it (Appendix F).

Categorical here is §2's law criterion — totals in the sampled vocal repertories — not a claim over every region and genre. Zarlino's discussion of the minor triad as the less perfect division of the fifth is contemporaneous (1558).

Stage two is what distinguishes the Major-ization from a drift. Had the major third simply arrived along with thirds in general, the Major-ization would amount to no more than "cadential tonics acquired thirds." Instead a modal note was added, established across a century, and then systematically overridden by an extra-modal note. Both positions of the two-chord cadential nucleus — the cadential pre-tonic and the cadential tonic — are now extra-modal.

**The law's instrumental extension, 1681–1714.** The trio-sonata repertoire carries the law into the eighteenth century: among closed minor endings, Corelli's Opp. 1, 3, 4, 5, and 6 (1681–1714) and Vivaldi's Op. 1 (1705) hold the Picardy third at 89.5% pooled (51 of 57; interval 78.9–95.1) — a small but real step below the Tasso totals (128 of 128; the difference is significant), and statistically indistinguishable from the chorale repertoire that follows (94.2%). The same repertoire shows something else, developed in §6: two-thirds of its minor-destination movements (117 of 174) close on a bare octave or fifth — the fourteenth century's third-less ending, standing throughout as the instrumental alternative to the choice the Major-ization had forced.

### The Fortification — the augmented sixth. Noise-level through 1650; established by c. 1740; still rising at 1800. Position: cadential pre-pre-tonic.

The augmented sixth belongs to no diatonic mode: flat sixth degree in the bass, raised fourth above the flat sixth, the two converging by contrary semitones onto the cadential pre-tonic. The augmented sixth is a double leading-tone to the cadential pre-tonic — an extra-diatonic chord serving an extra-modal chord.

Detection runs on distinct verticalities: a verticality is a maximal time slice during which the sounding pitch set does not change; a verticality carrying at least two distinct pitches is *eligible*, and eligible verticalities are the denominator throughout (Appendix C). Through 1650 the augmented sixth registers only at detection-noise level: **25 events in 1,128,193 eligible verticalities** across the four Renaissance corpora — 0.022 per 1,000 — with the chromatic madrigalists included, and with Palestrina contributing zero in 295,592 eligible verticalities. Event-level adjudication is decisive: of the 25 Renaissance events, 18 involve an accidental supplied by a modern editor rather than written in the source; only 5 resolve by converging semitones; not one falls in the final two percent of its piece; and

exactly one event is both scribal and resolving, and that one sits mid-piece. What the Renaissance corpora contain is therefore not a weak or early form of the Fortification but a different object — mostly an artifact of editorial accidental supply, rarely resolving, never cadential. The augmented sixth as a cadential device does not exist before the eighteenth century in any corpus sampled here.

By the Bach chorales the character inverts completely: 3 events in 30,067 eligible verticalities (0.100 per 1,000), all three scribal, all three resolving — and all three completing the full progression, augmented sixth to dominant to tonic, within a few verticalities, mid-piece. Then, with medium held roughly constant across four keyboard corpora: Scarlatti (c. 1740) 0.289 per 1,000 eligible verticalities, Haydn 2.087, Mozart 3.044, Beethoven 3.271, and onward to Chopin's preludes at 3.232 and mazurkas at 5.310. From Scarlatti to Beethoven the rise is roughly elevenfold, and monotone.

Three observations locate the augmented sixth.

**The augmented sixth is cadentially functional at its arrival — at phrase level.** Measured with a three-chord test (augmented sixth, then a dominant-quality sonority on the resolution note within four eligible verticalities, then a tonic-third arrival within eight more; Appendix C): at Scarlatti, 10 of 11 events proceed to the dominant-quality sonority and 6 of 11 complete the full chain to a tonic — every completed tonic minor. The chorale events complete 3 of 3. This is the sense in which *established by c. 1740* meets §2's criterion: the chord occurs, resolves, and serves the cadential progression from its first corpus onward. The completion rate then decays monotonically across the sonata corpora — 54.5% at Scarlatti, 38.5% at Haydn, 29.1% at Mozart, 19.2% at Beethoven — before partially rebounding in Chopin (25.0% in the preludes, 32.1% in the mazurkas), as the chord loosens from the progression that bore it into a free chromatic object.

**The augmented sixth appears enriched in minor and then generalises out of minor.** Classified by movement key (final-chord quality misclassifies repertoires whose minor movements end open, Scarlatti's above all; Appendix C), the rates per 1,000 eligible verticalities, minor-key against major-key movements: Scarlatti 0.597 against 0.174 — a 3.4-fold enrichment, on small counts of 7 and 4 events; Haydn 3.783 against 1.668; Mozart 7.785 against 2.585; Beethoven 2.958 against 3.168 — parity. The enrichment series is not monotone (3.4-fold, 2.3-fold, 3.0-fold, then parity), but its endpoints carry the claim: the chord enters as a minor-mode object, exactly where the Fortification's account places it, and within two generations reaches mode-neutrality. The Chopin corpora complicate rather than extend the trajectory: the preludes revert to strong minor enrichment (3.754 against 0.304, on a single major-key event) while the mazurkas sit near parity (5.632 against 4.596).

**The augmented sixth's bass note is the flat sixth degree** — the one cadence-adjacent degree that the Dominant-ization and the Major-ization left modal, and a degree that Ionian lacks and must borrow from minor.

**And the augmented sixth almost never closes a movement.** Beethoven's 407 events distribute across the ten deciles of movement-time as 47, 37, 16, 30, 74, 36, 45, 36, 52, 34, and exactly 1 of the 407 falls in the final two percent of its movement; across all 745 keyboard events measured here, 3 complete their three-chord chain inside a movement's final two percent. The fully rebuilt final cadence — augmented sixth, dominant seventh, major tonic, as one terminal gesture — existed as an available, describable cadence-type and essentially never as the actual final cadence. The Fortification armed the approach to cadences throughout the movement; §6 shows the conjunction never became the majority ending anywhere.

### The shape of the rise

**Spacing.** From the Dominant-ization to the Major-ization: at least a century and a half. From the Major-ization's completion to the Fortification's establishment: roughly **175 years**. Three changes across four

centuries. This is not one event with several symptoms.

**Order.** The three changes occupy three adjacent positions of the final cadence — the Dominant-ization the cadential pre-tonic, the Major-ization the cadential tonic, the Fortification the cadential pre-pre-tonic — but position is not the ordering variable: the sequence runs neither tonic-outward nor outer-inward, and the cadential tonic is rebuilt after the cadential pre-tonic. What orders the sequence is how Ionian-izing each change is. Audit the seven modes for each installed feature:

| Change | Feature installed | Modes natively possessing the feature |
|---|---|---|
| Dominant-ization | semitone below the final; in mature chordal form, a cadential pre-tonic of dominant-seventh quality | as scale degree: Ionian and Lydian; as chord: **Ionian alone** — Lydian's fifth-degree seventh chord is of major-seventh quality |
| Major-ization | major third at the cadential tonic | Ionian, Lydian, Mixolydian — three of seven |
| Fortification | augmented sixth at the cadential pre-pre-tonic | **none**, Ionian included |

The Dominant-ization installs Ionian's signature: the one cadential feature that identifies Ionian uniquely among the seven modes. The Major-ization installs a feature Ionian shares with two other modes. The Fortification installs a feature possessed by no mode at all: the Fortification makes the final cadence resemble Ionian's not one degree more, fortifying instead the approach to the already-rebuilt cadential pre-tonic. The sequence therefore runs in strictly decreasing order of Ionian-izing effect: the unique identifier first, the shared feature second, the feature Ionian itself lacks last. Position falls out as a corollary: the cadential pre-pre-tonic is rebuilt last, and empirically only after both nucleus positions — no fifteenth-century augmented-sixth practice exists, and none in a repertory whose cadential tonics are still minor.

## 5. Around the changes: the seven-mode field during the rise

**Part of the seven-mode field was removed rather than modified.** Lydian falls from 8.0% of works (composers born before 1440) to 2.9% to 0.0%, standing at 1.2% in 1571–1649; contemporaneously, Glarean (1547) files F-final pieces carrying a one-flat signature as transposed Ionian rather than as Lydian at all. Locrian never exceeds 0.5% in any reliable corpus (5 of 1,282 classifiable JRP works; 7 of 1,387 with the parser-recovered files included). Phrygian falls differently: 14.1% of sacred works against 2.9% of secular in the same period — Phrygian persists where repertory is bound to chant and has largely gone where repertory is not — and the semitone above the final that characterises Phrygian has a structural counterpart in common-practice minor: the iv$^6$–V half cadence, bass falling ♭6→5, approaching the cadential pre-tonic rather than the cadential tonic. The correspondence is structural, observed between two repertories; no documentary evidence is presented here that the later figure descends from the earlier one. The Fortification stands in the same structural relation, with a second semitone added beneath the first.

**The category proportions never moved.** Grouping modes by destination, the major/minor balance stands at 38.6/61.0 in 1420–1550 and 39.0/60.6 in 1571–1649. Ionian's own share of finals is likewise flat: 19.2% to 18.9% across two centuries. The rise did not move pieces onto C; the rise operated inside categories that stayed where the categories were.

## 6. The fall

### The Major-ization shattered — beginning by the 1710s, complete by the 1830s

The Picardy third at the cadential tonic of minor-mode movements, measured across the corpus groups (full table with intervals: Appendix D, Table D1):

| Corpus | Date | Picardy, closed endings | 95% CI |
|---|---|---|---|
| Tasso | 1580–99 | 128/128 = 100.0% | [97.1–100.0] |
| Tasso | 1600–19 | 38/39 = 97.4% | [86.8–99.5] |
| Corelli Opp. 1, 3–6 + Vivaldi Op. 1 | 1681–1714 | 51/57 = **89.5%** | [78.9–95.1] |
| Bach chorales | c. 1720–50 | 130/138 = **94.2%** | [89.0–97.0] |
| DCML keyboard and chamber | 1700–24 | 7/42 = **16.7%** | [8.3–30.6] |
| DCML | 1725–49 | 6/17 = 35.3% | [17.3–58.7] |
| Scarlatti sonatas | c. 1740 | 1/4 = 25.0% | [4.6–69.9] |
| Haydn + Mozart sonatas | c. 1760–90 | 1/8 = 12.5% | [2.2–47.1] |
| Beethoven sonatas | c. 1795–1822 | 6/19 = 31.6% | [15.4–54.0] |
| Chopin preludes + mazurkas | c. 1830–45 | 3/21 = **14.3%** | [5.0–34.6] |
| DCML | 1850–74 | 1/6 = 16.7% | [3.0–56.4] |
| DCML | 1875–99 | 3/17 = 17.6% | [6.2–41.0] |
| DCML | 1900–24 | 1/10 = 10.0% | [1.8–40.4] |
| DCML | 1925–49 | 0/10 = 0.0% | [0.0–27.8] |
| Hooktheory popular song | c. 1960–2020 | 366/2,778 = **13.2%** | [12.0–14.5] |

Four observations. **The collapse began inside the common practice, in the secular instrumental genres, while church-bound genres held.** The DCML 1700–24 bin — J. S. Bach's keyboard suites, Couperin's chamber concerts, Handel's keyboard music — sits at 16.7 percent, with minor-third finals outnumbering Picardy thirds five to one, in the same decades that the chorale repertoire holds 94.2 and the trio-sonata repertoire holds roughly 90. The split is not a comparison of distant composers: it runs through Bach's own generation, chorale against suite. **The collapse completed by the middle of the nineteenth century, and the level has not moved since**: the 1800–24 and 1825–49 bins still read 33.3 and 26.8 percent, settling thereafter; Chopin's 14.3 percent, the 0-to-18-percent floor of every quarter-century bin from 1850 to 1949, and modern popular song's 13.2 are one number within sampling error. **The instrumental repertoire also kept a third answer**: two-thirds of Corelli's and Vivaldi's minor-destination movements, sixteen of Scarlatti's twenty minor endings, and a third of Chopin's, close on a bare octave or fifth — the fourteenth century's ending, standing continuously available, declining the choice the Major-ization had forced. **And the collapse's shape is a law's shape**: from totals to a floor inside roughly a century, with no stable intermediate plateau — the signature, argued in Part II, of a rule that had two states rather than a sliding preference.

### The Dominant-ization bent

The raised-seventh share of seventh-degree notes in minor-destination pieces: 12.3 to 15.3 percent as notated in 1560–1649; **12.4 percent** (7,467 of 60,214) in modern Aeolian popular song. One unit difference must be disclosed: the modern figure is measured on melody lines (Appendix C), where the Tasso figure counts all voices, so the comparison is of magnitudes across similar but not identical

measures — both notated in their sources, which is what makes it sound at all. On that comparison, four centuries of style change — the seconda pratica, the galant, Romanticism, the collapse of common practice, the rise of recorded popular music — left the modern rate sitting inside the notated Tasso band of 12.3–15.3 percent, within three points of its top. Modern songwriters raise the seventh where the raised seventh points somewhere, at 12.4 percent of opportunities, much as Monteverdi's contemporaries did. (The fifteenth-century rate cannot join this comparison: it is roughly 95 percent editorially supplied, and falls to 0.0–0.4 percent when editorial accidentals are removed; Appendix E.)

### The Fortification rose through the fall

Between 1740 and 1820 the Major-ization fell from 94 percent toward 14 while the Fortification rose roughly elevenfold, from 0.289 to 3.271 events per 1,000 eligible verticalities; from Scarlatti forward the two series run in the same keyboard corpora. The De-Ionian-ization of the cadential tonic and the continued arming of the cadential pre-pre-tonic ran concurrently. The Fortification's modern fate is not measured here, and cannot be measured in the Hooktheory corpus: Hooktheory's annotators encode chords by sound, an augmented sixth is enharmonically identical in sound to a dominant seventh, and the spelling that defines the augmented sixth does not survive Hooktheory's sound-based encoding.

### After the release

Modern popular song, 26,175 entries with modes declared by the annotators: Ionian 50.4%, Aeolian 39.3%, Mixolydian 3.9%, Dorian 3.8%, Phrygian 1.1%, Lydian 1.0%, Locrian 0.3%, with 0.3% of songs declaring a scale that matches no diatonic mode.

Aeolian returned, declared as Aeolian at two-fifths of the repertoire, and the release divided into three facts. The Dominant-ization persists, unchanged. The Major-ization reversed completely. And the category structure did not reverse at all: Dorian, the single most common mode in 1500 at 37 percent, stands at 3.8; Phrygian at 1.1; Lydian at 1.0; Locrian at 0.3; Ionian and Aeolian together hold 89.7 percent. The pruning of the seven-mode field — the part of the Ionian-ization that operated by removal rather than by grafting — outlived the grafts by centuries and shows no reversal in the modern corpus measured here, a single pooled corpus that cannot by itself establish a trajectory.

## 7. The teaching record

The teaching literature is a corpus written by the Ionian-ization's own witnesses, and the teaching literature's dates can be laid against the corpus dates at every transition.

**The rise's earliest pedagogy states the Dominant-ization as conditional.** The fifteenth-century rubric *causa pulchritudinis* — an inflection for beauty — names an option, and the rubric names the option a century after the fourteenth-century practice the rubric describes (Bent 2002, 78–79; Hughes 1972). From that rubric to the modern rule "raise the sixth and seventh ascending," the teaching literature always gave the raised seventh a condition, and never stated the raised seventh as a law.

**The Major-ization's rationale arrives mid-transition.** The corpus transition runs c. 1540–1565; Zarlino's account of the minor triad as the less perfect division of the fifth is 1558 — theory rationalizing a change already half complete.

**The law's prescriptive era speaks in Simpson.** Simpson's *Compendium of Practical Musick* (1667) treats the major third at the close of a flat-key piece as standing practice and legislates only the third's voicing, judging "the sharp 3d more proper for an inward Part at Conclusion" — advice embedded in his four-part scoring instructions (Part II, §10), a teaching text written inside the law's window, presupposing the law rather than instituting the law.

**The Major-ization's obituary arrives a generation after the leading edge abandons the Major-ization.** The leading-edge collapse shows in the keyboard and chamber repertoire of 1700–1724; Rousseau's *Dictionnaire de musique* (Paris, 1768; the volume dated 1767) coins "tierce de Picardie" — and coins the term, by Rousseau's own account, "par plaisanterie," as a joke. Rousseau reports that ending minor pieces on the major chord was formerly a law — "on se faisoit autrefois une loi de finir toujours sur ce premier" — and that the usage lasted longest "dans la Musique d'Église, & par conséquent en Picardie," in church music, and therefore in Picardy with its many singing cathedrals (pp. 514–15; Waring's 1779 translation, p. 436: "it was formerly a law"). Rousseau's entry names the Picardy third at the moment of the Picardy third's obsolescence, attests the Picardy third's former status as a law in that exact word, and locates the Picardy third's last stronghold in church music — the same church-against-secular split the corpora measure in his century's first quarter.

**And the dead law keeps teaching.** Crotch's *Elements of Musical Composition* (1812) still prescribes the practice — the minor-key tonic "is generally made a major triad," with the Picardy name relegated to a historical footnote — a textbook teaching the major ending as the general rule a full century after the secular leading edge abandoned it. When the topic settles into the standard English textbooks it carries Rousseau's church localization with it: Ouseley (1868) files the Tierce de Picardie among the cadences, and Prout (1889, §193) reports it "still not infrequently employed, especially in church music" — the 1768 observation, republished as current fact in 1889. Twentieth-century textbooks split the framing: Foote and Spalding historicize the device (1905, retained verbatim through their 1936 revision — listeners "a century or so ago" felt the surprise; "it was much used by Bach"), Kitson assumes it as elementary knowledge (1914), and Hull re-presents it as a forward-looking resource of the modal revival (1915). The German textbook lineage — Weber, Marx, Richter, Jadassohn, Riemann — never teaches the term at all: the named topic is an English-and-French lexical tradition laid over a practice the whole of Europe had once enforced.

**The Fortification's codification trails the Fortification's establishment, and the classroom labels trail by two generations more.** Composers establish the augmented sixth by 1740; Marpurg derives the augmented sixth as an altered tertian harmony in the *Handbuch bey dem Generalbasse und der Composition* (1755, 125–27; 2nd ed. 1762, 45–46); Rameau declares, in the *Lettre à M. d'Alembert* published with the *Code de musique pratique* (1760, 6–7), that the interval "n'est susceptible de B.F. ni de renversement" — admits neither fundamental bass nor inversion; Kirnberger treats the diminished third and its inversion to the augmented sixth in *Die Kunst des reinen Satzes* (1771–79, I:21–25); Vogler (*Handbuch zur Harmonielehre*, 1802, 101–110) and Gottfried Weber (*Versuch einer geordneten Theorie der Tonsetzkunst*, 1817–21) systematize the altered-tertian and enharmonic account after 1800. The Italian, French, and German labels arrive last: the earliest verifiable appearance of the three-name set is Callcott's *A Musical Grammar* (1806, Arts. 441–443), which reports "Italian sixth" as already current, coins "French" for the form "only found in the Theory of Rameau," and coins "German" for the form found "in the writings of Graun" — a classroom taxonomy whose geographic names, as Ellis (2010) concludes, originated in the early nineteenth century and "reflect only the vaguest of actual national origins."

**The magnification is measured in the repertoire and documented as a teaching lineage.** The chorale repertoire — church-bound, pedagogy-adjacent — holds the Major-ization at 94.2 percent in the same quarter-century that the keyboard and chamber repertoire holds it at 16.7. That most Major-ized repertoire then entered the classroom by a chain that is documented at both ends and at every join. It begins in Bach's own teaching: C. P. E. Bach's 1775 account of his father's method has pupils writing alto and tenor to chorales whose outer voices Bach supplied. Kirnberger's *Kunst des reinen Satzes* (1771, I:157) prints the same exercise and names Bach's chorales "the best models for diligent study" — and the same Kirnberger pressed for the Breitkopf edition, which appeared in 1784–87, after his death. Becker packages 69 chorales explicitly as figured-bass exercises (1832); Marx's composition method compares

Bach settings as chorale-harmonization models (1837); and by 1850 Erk's preface protests that the chorales have become "ordinary exercise-pieces for thoroughbass students" — the complaint that proves the routine. The Leipzig Conservatory's official harmony text (Richter, 1853) makes chorale harmonization the central exercise; conservatory texts assign Bach's chorales explicitly from the 1880s (Goetschius for Stuttgart, 1882; Chadwick for the New England Conservatory, 1897, printing seventeen Bach chorales as an appendix); Prout's revised *Harmony* (1901–03) brings the practice to the British mainstream, telling students that "in the whole of Bach's Chorals, numbering very nearly four-hundred, not more than about half a dozen end on a minor chord"; and Riemenschneider's Schirmer "371" (1941) codifies a classroom practice by then a century and a half old — an edition chain running Dietel (c. 1735), Birnstiel (1765/69), Breitkopf (1784–87), Becker, Erk, Richter (1898), Terry (1929, the earliest verified English-language edition), Riemenschneider (1941). Two bounds hold the claim honest: the lineage is German-then-Anglo-American, not universal — Cherubini, Day, Macfarren, Jadassohn, and Riemann never use chorale models, and in Paris chorale harmonization appears only in d'Indy's rejected 1892 reform proposal, the Conservatoire keeping to its basses and chants donnés — and how many students each institution put through the exercise remains uncounted, the syllabus-level records being archival (§13). Within those bounds, the fact stands: textbooks still teach the Picardy third from Bach's chorales while composers have used the Picardy third in roughly one closed minor ending in seven for two centuries.

**The taxonomy finishes last of all.** The three-forms filing of minor is a nineteenth-century construction completed in the twentieth. Though "natural minor scale" appears in English as early as the 1820s (Goodban 1820, 32; Relfe 1821, 32) and named the unaltered scale in mid-century American singing manuals (Mason 1854, §72; Root 1855, §72 — "natural" against "artificial" minor), the century's standard harmony textbooks filed that scale as the "Aeolian" or descending "melodic" form (Prout 1889, §§169–172; Goetschius 1892, §88; Foote and Spalding 1905, §64); Prout's 1889 taxonomy — one harmonic and two melodic minor scales — already files the three forms without the modern name. "Natural minor" became the ordinary classroom name only in the twentieth century, its printed frequency beginning a sustained climb in the 1930s and accelerating after 1960 (Google Books Ngram, English corpus; Appendix D, Table D8) — three centuries after the practice that the three-forms scheme files.

The summary fact, stated once: at every measured transition — the rubric after the practice, Zarlino mid-rise, Rousseau after the collapse, Marpurg through Weber after the establishment, the taxonomy last — the written record moves a generation or more behind the composed record, and Simpson writes from inside the Major-ization's window, presupposing rather than instituting. In none of the sources examined does a prescription precede the practice the prescription describes; the Purcell-revised Playford (1694) remains unexamined and stands as a named test (§13).

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# Part II — The explanation

## 8. What any explanation must satisfy

Part I is agnostic about causes. Part I's shape is not. Six constraints fall out of the data.

**C1 — Seriality of the rise.** Three changes across four centuries, roughly 175 years between the Major-ization's completion and the Fortification's establishment. Any single-event cause — a taste shift, a generational turn, a technological moment — is excluded.

**C2 — Ordering by Ionian-izing effect.** The three changes arrive in strictly decreasing order of Ionian-izing effect: the unique identifier, then the shared feature, then a feature no mode possesses. An adequate explanation must produce that gradient. The gradient also carries the positional dependency:

the least Ionian-izing change, whose function is to approach the rebuilt cadential pre-tonic, appears only after the nucleus is rebuilt.

**C3 — Dependency structure.** One apparent change is not independent: the raised sixth arrives as the raised seventh's companion, the conditional dependence robust under every editorial-accidental policy (§4). An adequate explanation must distinguish changes from consequences — must say why the raised seventh's melodic repair follows in the same breath while the cadential tonic's rebuilding waits a century and a half, and the cadential pre-pre-tonic's rebuilding another hundred and seventy-five years.

**C4 — Three-way reversibility.** The De-Ionian-ization split the three outcomes: the Major-ization shattered (totals to a 14-percent floor, the floor then flat for two centuries); the Dominant-ization bent (three points in four centuries); the emptied categories have not refilled in any corpus measured here (Dorian 37 percent to 3.8). Perception did not change in 1700, and perception did not change in 1950. Any purely perceptual explanation predicts the changes rising together and falling together, and the changes did neither.

**C5 — Concurrency of rise and fall, split by genre.** In the same decades — the first half of the eighteenth century — the Picardy law held at ceiling in church-bound repertoire, had already collapsed in secular keyboard and chamber repertoire, and the Fortification was rising in that same secular repertoire. Any account that treats the period as a single motion — elaboration, simplification, decline — must explain why the same years moved two adjacent cadential positions in opposite directions, and moved the same position in opposite directions across genres.

**C6 — The lag.** The written record trails the composed record at every transition measured here — usually by a generation or more, at the nearest writing from inside a change's window (Zarlino mid-transition; Simpson inside the law's window) — and in the sources examined no prescription precedes the practice the prescription describes. Any explanation in which theory or teaching drives the changes is excluded — and any explanation that appeals to written rules must locate the operative rule in practice, with the writing as the rule's record rather than the rule's engine.

## 9. Candidates that fail the constraints

**(a) A taste shift toward major sonority.** Fails C1, the changes being serial rather than simultaneous; fails C2, since a preference for major thirds has no reason to reach the cadential tonic a century before the cadential pre-pre-tonic; fails C5, since a taste for major cannot rise and fall at adjacent positions at once, still less split by genre. Retained only as a possible local contributor to the Major-ization.

**(b) Perceptual superiority.** Huron's derivation of voice-leading norms from auditory principles (2001, 2006) plausibly explains why the operative rules were what the rules were — why semitones pull, why the tritone was policed. As a direct cause of the rise and fall, perception fails C4: hearing is constant, and the fall split the three outcomes along a line hearing does not draw. Perception belongs underneath the explanation rather than in the explanation.

**(c) Vernacular assimilation.** Glarean's account (1547) presents the added modes as already current outside the church repertory; perhaps learned practice merely caught up with popular practice. Vernacular assimilation satisfies C1, being a standing pull, but not C2 — nothing in vernacular priority explains the Ionian-izing gradient — and vernacular assimilation is silent on C3, C4 and C5. The corpora give vernacular practice a limited but real role: secular repertory led on subtraction, having shed Phrygian (2.9 percent secular against 14.1 sacred) long before sacred music did, and secular instrumental genres led the Picardy collapse by decades. A contributing current, not the mechanism.

**(d) Theory-driven change.** Perhaps the treatises taught the changes and composers complied. C6 excludes theory-driven change outright: the treatises trail the composers at every transition, and one cannot cause what one follows. The teaching record's actual role appears under §10.

## 10. The answer: the fixed-point conjecture

Polyphonic practice operated under two maintenance rules, applied by singers wherever cadences were made: inflect to remove the tritone against the final, and inflect to supply the cadential semitone. The rules are practiced rules; the treatises that name the rules — *causa necessitatis*, *causa pulchritudinis* — are the rules' record, written a century after the practice, exactly as C6 requires. Jointly the two rules specify one diatonic configuration — raised seventh present, tritone-with-the-final absent — and exactly one mode satisfies both without alteration: Ionian, whose final cadence, once seventh chords standardize, is V7–I. The conjecture: **these standing practiced rules carried every mode's final cadence toward that one stable configuration; the rise of §4 is the record of the carrying; and the fall of §6 is what the carried structure did when the styles enforcing each part of the structure changed, each part failing according to each part's own kind.**

Against the constraints:

The conjecture satisfies **C1** because the rules were standing — applied at every cadence, for centuries — a constant force producing serial change as each solution's residue became the next target: the rebuilt cadential pre-tonic leaves the cadential tonic mismatched, and the cadential tonic is made major; the rebuilt cadential pre-tonic stands unprepared, and a cadential pre-pre-tonic is built for the rebuilt cadential pre-tonic.

The conjecture satisfies **C2** because the two rules, taken jointly, are a specification of precisely the feature that is unique to Ionian: the cadential semitone rule commands the raised seventh, the tritone rule requires the perfect fourth, and those two degrees are exactly the tendency-tone pair of the dominant seventh. The rules do not describe Ionian and command imitation; the rules command two inflections whose joint product is the one cadential machine only Ionian possesses natively. The Dominant-ization is the rules executed — which is why the most Ionian-izing alteration comes first: the Dominant-ization is the only alteration the rules directly command. The remaining changes are consequent rather than commanded, each harmonizing another cadential position with what the Dominant-ization installed, and each adding less resemblance than the last because the identifier is already in place — the cadential tonic matched to the rebuilt pre-tonic (the Major-ization), the rebuilt pre-tonic given a preparation (the Fortification), a preparation that Ionian never needed and cannot natively build.

The conjecture satisfies **C3** because the conjecture distinguishes two kinds of consequence. The raised sixth repairs a defect internal to a single line — an unsingable interval — and a line either works or does not work, so the repair follows in the same breath as the inflection it repairs. The Picardy third and the augmented sixth repair defects in the relation between chords, and a relation between chords becomes audible as a problem only after the surrounding harmonic apparatus develops. Immediate melodic repair, delayed harmonic repair: the conjecture predicts the dependency structure observed.

The conjecture satisfies **C4** because the three outcomes had three structures, and structure fixes fate. The Major-ization had hardened into a **law**, and a law has two states: while the enforcing style holds, compliance is total, because the alternative has stopped being available to a competent practitioner — as a minor final chord had stopped being available between 1565 and 1700, in the way parallel fifths had stopped being available. When the enforcing style loosens, no seventy-percent version of the law exists to retreat to; "always" becomes "composer's choice" in one step, and the rate falls immediately to wherever taste sits — totals at ceiling, then a fall to a floor, then a floor flat for two centuries, the endpoints coinciding. The Dominant-ization was always a **choice** — *causa pulchritudinis* names a condition, not a duty — and a choice has a continuum of states, so a choice has nothing to shatter: a rule of the form "apply this where the condition holds" survives any change of style that leaves the condition still occurring somewhere, which is why four centuries moved the raised-seventh rate by three points. And the emptied **categories** did not refill anywhere in the measured record, the conjecture's proposed reason being that a category, once emptied, has no condition under which the category refills: laws

shatter, choices bend, and categories stay empty. Perception explains none of the three curves; the three conventions' statuses explain all three curves.

The conjecture satisfies **C5** because the two concurrent motions served different cadential positions, and because a law's enforcing style is a style, not a calendar: the galant loosening reached the secular keyboard and chamber genres first, and the Picardy law broke there first — by the 1710s — while the church-bound genres, where the enforcing style still held, kept the law at ceiling for another generation, exactly the split Rousseau reports from memory in 1768 when he locates the law's last stronghold in church music. The Fortification, meanwhile, served the still-standing Dominant-ized cadential pre-tonic, and spread with the same secular styles that broke the Major-ization. One structure falling while another structure rises, and one structure falling in one genre while holding in another, is exactly what independent conventions with different enforcing styles do — and is inexplicable to any account that insists the era had one direction.

The conjecture satisfies **C6** by construction, and C6 in turn assigns the teaching literature the teaching literature's real role: a trailing mirror and a magnifier. The treatises record the practiced rules a generation late, encode each convention in the kind — law or choice — that the convention's corpus curve independently displays, and magnify whatever repertoire the treatises carry: the chorale corpus, embalmed as teaching material from the 1760s onward through an edition chain that runs unbroken to the present (§7), carried the shattered Major-ization into classrooms for two centuries. The teaching literature is evidence throughout, and engine nowhere.

**Objections, briefly.** *Ionian's share of finals never grew, so nothing converged*: the sequence operates on cadential content, below the level a finals-count can detect — at final cadences the modes differ sevenfold in repair-need (Ionian 63.9% requiring no inflection against Aeolian's 8.5%) while the finals stay put; rebuilding in place is the observation, and migration would have indicated abandonment rather than convergence. *Aggregate cadence measures show no trend*: correct and expected — the repair-free rate computed over all cadences stays within a narrow, trendless band (roughly 19–28 percent across cohorts) because cadences landing away from a piece's final are mode-neutral and far outnumber mode-confirming cadences; nothing here pools the two. *The modes may not have been compositionally real (Powers)*: the finding is measured on sounding pitch content — thirds in cadential tonics, raised degrees, augmented-second occurrences, augmented-sixth simultaneities — and the conjecture invokes practiced rules, which are documented operations whatever name the operated-on objects carry; the objection dissolves a vocabulary, and the sequence and the mechanism survive in any vocabulary. *Editorial ficta contaminates the counts*: the strongest technical objection, and the reason every accidental-dependent series in this paper is reported under explicit editorial policies (Appendix E), with the argument resting on the measures robust to the policies — written cadential thirds, spelled augmented-sixth simultaneities, and conditional dependences that survive the scribal-only accounting.

**What the conjecture does not claim, and cannot exclude.** The conjecture does not claim intention: nobody aimed at Ionian, which had no name in church theory before 1547. The conjecture does not claim the rules were arbitrary: the rules' perceptual grounding (§9b) and vernacular reinforcement (§9c) are compatible substrates, and the true history is most likely the fixed-point mechanism running on perceptual substrate with vernacular currents alongside. What the conjecture cannot exclude is a hybrid in which vernacular practice supplied the direction and the practiced rules merely ratified vernacular practice; separating the hybrid from the conjecture requires corpus work on secular monophony before 1500 — mode-and-cadence rates in the trouvère, Minnesang and early song repertories — which does not yet exist in analyzable form (§13).

**What would falsify the finding.** Any of: the changes proving simultaneous under better dating; the Ionian-izing gradient inverting — a repertory establishing Picardy thirds before raised sevenths, or augmented sixths while the repertory's cadential tonics remain minor; a sixteenth-century augmented-

sixth practice; the 1540–65 transition dissolving into a three-century drift; a dated corpus of closed minor endings between 1720 and 1830 holding the Picardy third at ceiling in a secular genre; a mainstream composition textbook after 1780 prescribing the major final third as exceptionless; a modern corpus with declared modes showing the raised seventh collapsed toward zero; or a prescription anywhere in the record that precedes the practice the prescription describes.

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# Part III — Implications, open work, and conclusions

## 11. Two scales that are not scales

If §4's account is right, a corollary follows for the objects modern pedagogy teaches as the three forms of minor, and the corpora can test it.

**Harmonic minor's defining interval does not occur.** A scale, taken as a melodic object, is an ordered succession of steps a line may traverse, and harmonic minor's characteristic step — the augmented second between the flat sixth and the raised seventh — is the interval that distinguishes harmonic minor from every other scale. Melodic occurrences of that step, in Aeolian and Phrygian pieces: **1** in 341,005 notes (JRP), **0** in 153,963 (Palestrina), 7 in 76,445 (Tasso). In the vocal repertories sampled here the characteristic step is essentially never traversed, so within those repertories harmonic minor does not operate as a scale. The measurement covers Renaissance and early-Baroque vocal corpora only; what harmonic minor is in later composition, in instrumental writing, in pedagogy, or as a theoretical construct is outside what these corpora settle. What the corpora do show is what harmonic minor describes: raise the seventh inside the cadential pre-tonic, leave the sixth alone in the subdominant chord, and write the contents of both chords on a single staff as though the contents formed a line. The augmented second is not a step; the augmented second is the gap between two chords, which is why the corpora show the augmented second all but uncrossed. The result is robust in the conservative direction: editors supplying ficta tend to create augmented seconds rather than suppress them, and the observed count is near zero regardless.

**Melodic minor's rule names the wrong variable.** For every sixth-degree note in Aeolian and Phrygian pieces, the raised-sixth rate, split by direction and by proximity to a raised seventh within two notes (full cells and window sensitivity: Appendix D, Tables D2–D3):

| Corpus | ASC, near ♯7 | ASC, far | DESC, near ♯7 | DESC, far |
|---|---|---|---|---|
| JRP c. 1420–1550 | **87.4%** (n=223) | 4.0% (n=15,147) | — (n=20) | 0.6% (n=29,902) |
| Palestrina c. 1554–94 | **93.9%** (n=507) | 18.0% (n=5,932) | **77.4%** (n=328) | 10.3% (n=12,922) |
| Tasso 1571–1649 | **79.7%** (n=128) | 22.1% (n=2,686) | **37.0%** (n=73) | 4.7% (n=6,205) |
| Bach chorales c. 1720–50 | **87.5%** (n=272) | 33.1% (n=1,306) | 28.8% (n=80) | 13.2% (n=2,665) |

Holding direction constant, proximity multiplies the rate by 22×, 5.2×, 3.6× and 2.6×. In Palestrina, descending lines beside a raised seventh take the raised sixth 77.4% of the time — four times the rate of ascending lines that are not near a raised seventh — and Tasso shows the same crossing (37.0% against 22.1%); in the Bach chorales the descending-near cell stays below ascending-far (28.8% against 33.1%), and JRP's descending-near cells are too small to evaluate, so the cross-comparison is carried by Palestrina and Tasso while the within-direction multipliers hold everywhere (Appendix D, Table D2). Direction is not the governing variable; adjacency to the raised seventh is — governing in the predictive sense: the cells establish association and predictive superiority over the direction rule, not the cognitive mechanism behind the association. In Dorian, whose major sixth is modal, the same measure reads high (roughly 59–81%) and unconditioned in every period: where the raised sixth is the scale's own degree the

measure saturates, and where the raised sixth is extra-modal the measure tracks the raised seventh. Raised sevenths are usually approached from below, so "near a raised seventh" and "ascending" correlate heavily, and a rule stated in terms of direction reproduces most of the observed behaviour — which is sufficient to explain why a direction-based rule remains serviceable in pedagogy. Whether the pedagogical rule arose by mistaking the correlate for the cause is a claim about transmission, and no documentary evidence for that transmission is presented here.

**What the two scales are.** Both describe the same thing from two angles: the region around the raised seventh. Harmonic minor is the Dominant-ization described vertically; melodic minor is the Dominant-ization described horizontally. Nineteenth-century pedagogy inherited a repertory saturated with extra-modal raised sixths and sevenths, needed to teach that repertory, and scales are what pedagogy teaches — so, on this reading, offered as interpretation rather than documented transmission, pedagogy produced two: one scale for the harmony, one for the melody. In the repertories sampled here neither operates as a scale, and harmonic minor contains a step those repertories forbid. The three-forms taxonomy — assembled in the nineteenth century, named in the twentieth (§7) — is a filing scheme laid over an accumulation, and the filing scheme files one change twice. (Full analysis: the companion study *Two Scales That Are Not Scales*.)

## 12. The coda the sequence wrote

One structural observation, offered as an observation. The sequence has the shape of the final cadence the sequence built. Each change resolves a tension the previous change created, and creates the tension the next change resolves — and does so, as a cadence does, on two time-scales at once. The raised seventh leaves an unsingable interval hanging in the line, and that interval is resolved immediately, in the same breath, the way an appoggiatura resolves. The raised seventh also leaves the cadential tonic mismatched to the rebuilt pre-tonic, and that mismatch is resolved a century and a half later; and the raised seventh leaves the rebuilt pre-tonic unprepared, and that preparation waits another hundred and seventy-five years. Immediate resolution and long suspension, running simultaneously: the near tension settled at once, the far tensions held across generations.

Then the sequence stops. After the Fortification, no fourth change appears in the corpora sampled here. The Ionian-ization, after four hundred years, ends at the moment the final cadence is completely rebuilt — rebuilt, in the end, past Ionian itself: minor's final cadence closes more heavily armed than Ionian's own, fortified by a chord that Ionian must borrow from minor to use. And a cadence is precisely that: motion whose function is to end motion.

The release did not wait for the arrival. While the Fortification was still rising, the Major-ization was already falling — at ceiling in the chorales, collapsed in the suites of the same decades, 14 percent by Chopin — a tone of the resolution sounding before the final chord, which music calls an anticipation. The Major-ization fell away, the Dominant-ization remained the choice the Dominant-ization had always been, and Aeolian — never destroyed, only rebuilt at Aeolian's edges — was simply there again, as Aeolian had been all along beneath the changes. The fermata lifted. The music moved on.

## 13. What cannot be settled from a screen

The measurements above run on every machine-readable corpus this study could obtain, and the citations rest on every source page a digitized copy would show. What remains is the work that no amount of computation reaches, listed here as the study's open flank.

**Archives and libraries.**

- **Syllabus-level curriculum records.** The teaching lineage of §7 is documented in textbooks and editions; how many students each institution put through the chorale exercise, in which decades, is not. The digitized conservatory prospectuses turn out to be genre-blind — the Royal Academy's near-

annual series (1880–2020) names harmony as a subject and its textbook-author professors on staff in every decade, but never names classroom materials — so the count lives in syllabi, examination papers, and class registers: the Leipzig Conservatory's annual reports, the Paris Conservatoire's class records beyond Pierre's 1900 documentary history, and their Anglo-American counterparts, all archival.

- **Page-level verification still open.** Berger (1987) at page level for the necessity/beauty chapters; Vogler (1802) and Weber (1817–21) at page level from the primary prints; the 1722 printing of Simpson, a reissue whose pagination must be checked against the copy itself; Piston's *Harmony* (1941) and the preface of Riemenschneider's 1941 edition, both lending-locked in every digital copy reached; and any use of "Italian sixth" in the London thoroughbass tutors of 1780–1806, which requires the eighteenth-century short-title archives.
- **The Purcell-revised Playford (1694)** — the named test of §7: does any prescription there precede practice?

**Editing and encoding.**

- **Secular monophony before 1500.** The trouvère, Minnesang, and early song repertories exist in editions but not in analyzable encodings; without them the vernacular-priority hybrid of §10 cannot be separated from the fixed-point conjecture. This is an encoding project measured in years, not a query.
- **The vocal repertories of 1650–1720.** The instrumental bridge is measured here; the sacred and secular vocal music of the same decades — where the Picardy law's genre split first opened — is largely unencoded.
- **Corelli's Opus 2**, absent from the encoded set, and the expansion of expert-annotated corpora across 1850–1950, where the measured bins are thin.

**Ears and instruments.**

- **Expert cadence annotation.** The cadence detector's recall limits are measured against its own criteria (Appendix F); its precision against human judgment is not. A musicologist-annotated cadence sample — ears on scores — is the missing calibration.
- **Performance practice.** How much *musica ficta* performers actually sang, against what scribes wrote and editors supplied, is a question about performance, approachable through surviving instructions, intabulations (instrumental arrangements that fix accidentals the vocal sources leave open), and reconstruction — not through counts alone.

## 14. Conclusions

Three datable changes, arriving in decreasing order of resemblance to Ionian's cadence, rebuilt the final cadence of minor-mode music between the fourteenth century and the eighteenth; the rebuilt structure then came apart along three curves whose shapes match the three conventions' kinds — the law shattered, the choice bent, the emptied categories stayed empty. The collapse of the law began in secular instrumental genres by the 1710s while church genres held it at ceiling, and the level it fell to has not moved in two centuries. The teaching record trails the composed record at every measured step, magnifying a church repertoire embalmed at the moment of its own obsolescence, and completing its taxonomy — two scales that describe one change — three hundred years after the practice the taxonomy files. A pair of practiced rules, applied at every cadence for centuries, suffices to generate the whole sequence, its ordering, its dependency structure, and its three-way unwinding; nothing else examined here does. The conjecture is falsifiable at a dozen named points, and the work that would test it next — in archives, in unencoded repertories, and by ear — is stated. The history of minor's cadence is the history of

a fixed point: approached for four centuries, reached, fortified, and then released — each part by its own kind of letting go.

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## Appendix A — Sources

*Primary and pedagogical, with verified locations.* Marchetto da Padova, *Lucidarium in arte musice plane* (1317–18), tract. 2 capp. 4–8 (the fivefold division of the tone; the four-diesis chroma) and tract. 5 cap. 6 (discant application); ed. and trans. Jan W. Herlinger, *The Lucidarium of Marchetto of Padua* (University of Chicago Press, 1985), 141–43, 148–55, 208–23; see also Herlinger, "Marchetto's Division of the Whole Tone," *JAMS* 34/2 (1981): 193–216. The *causa necessitatis / causa pulchritudinis* tradition: Margaret Bent, "Musica Recta and Musica Ficta" (1972), repr. in *Counterpoint, Composition, and Musica Ficta* (Routledge, 2002), ch. 1, at 78–79; Karol Berger, *Musica Ficta: Theories of Accidental Inflections in Vocal Polyphony from Marchetto da Padova to Gioseffo Zarlino* (Cambridge University Press, 1987), Part II; Andrew Hughes, *Manuscript Accidentals: Ficta in Focus 1350–1450*, Musicological Studies and Documents 27 (American Institute of Musicology, 1972). Glarean, *Dodecachordon* (1547). Zarlino, *Le istitutioni harmoniche* (1558; rev. 1589). Christopher Simpson, *A Compendium of Practical Musick* (1667), Part II §10 (the "sharp 3d… inward Part" judgment; p. 45 in the 1732 printing). Jean-Philippe Rameau, *Traité de l'harmonie* (1722); *Code de musique pratique* with the *Lettre à M. d'Alembert* (1760), Lettre at 6–7 (the augmented sixth "n'est susceptible de B.F. ni de renversement"), Code at 124–25. Jean-Jacques Rousseau, *Dictionnaire de musique* (Paris, 1768; volume dated 1767), art. "Tierce," sub-entry "Tierce de Picardie," 514–15; trans. William Waring, *A Complete Dictionary of Music*, 2nd ed. (1779), 436. F. W. Marpurg, *Handbuch bey dem Generalbasse und der Composition* (1755), 125–27; 2nd ed. (1762), Abschn. I Abs. III, 45–46. J. P. Kirnberger, *Die Kunst des reinen Satzes in der Musik* (1771–79), I:21–22, 24–25. G. J. Vogler, *Handbuch zur Harmonielehre* (1802), 101–110. Gottfried Weber, *Versuch einer geordneten Theorie der Tonsetzkunst* (1817–21; 3rd ed. 1830–32, I:259–60). John Wall Callcott, *A Musical Grammar, in Four Parts* (1806), Part III ch. VI sect. II, Arts. 441–443 (the national labels). C. P. E. Bach, ed., *Vierstimmige Choralgesänge* (Birnstiel, vol. 1, 1765; vol. 2, 1769, seen through the press by Agricola and disavowed by C. P. E. Bach) and *Vierstimmige Choralgesänge* (Breitkopf, 1784–87; prepared by Kirnberger's advocacy, published after his death); the chorale edition chain: Dietel manuscript (c. 1735), Becker (Breitkopf, 1831–32, with 69 chorales with figured bass; Friese, 1841–43), Erk (Peters, 1850, 1865; preface of 8 December 1850), Dörffel (1871), Bach-Gesellschaft vol. 39 (1892), Richter (1898), Terry (Oxford, 1929), Smend (1932), Riemenschneider (Schirmer, 1941). C. P. E. Bach's account of J. S. Bach's teaching: letter to Forkel, 1775 (in *The New Bach Reader*). Teaching texts: William Crotch, *Elements of Musical Composition* (1812); A. B. Marx, *Die Lehre von der musikalischen Komposition* (1837; trans. Saroni, 1854); E. F. Richter, *Lehrbuch der Harmonie* (1853; trans. 1867); F. A. G. Ouseley, *A Treatise on Harmony* (1868); Percy Goetschius, *The Material Used in Musical Composition* (1882, for Stuttgart); G. W. Chadwick, *Harmony: A Course of Study* (1897, New England Conservatory); C. H. Kitson, *The Evolution of Harmony* (1914); A. Eaglefield Hull, *Modern Harmony* (1915); Constant Pierre, *Le Conservatoire national de musique et de déclamation* (1900), including d'Indy's 1892 *Projet d'organisation des études*. Thomas Goodban, *The Rudiments of Music* (1820), 32; John Relfe, *Lucidus Ordo* (1821), 32; Lowell Mason, *The Hallelujah* (1854), §72; George F. Root, *The Young Men's Singing Book* (1855), §72; John Stainer and W. A. Barrett, *A Dictionary of Musical Terms* (1876), s.v. "A"; Ebenezer Prout, *Harmony: Its Theory and Practice* (1889), §§169–172; Percy Goetschius, *The Theory and Practice of Tone-Relations* (1892), §88; Arthur Foote and W. R. Spalding, *Modern Harmony in its Theory and Practice* (1905), §64.

*Scholarship.* Bent (2002); Berger (1987); Powers, "Tonal Types and Modal Categories" (1981) and "Is Mode Real?" (1992); Meier (1974/1988); Dahlhaus (1968/1990); Lester, *Between Modes and Keys* (1989); Christensen (1993); Dodds, *From Modes to Keys in Early Modern Music Theory* (2023); Huron,

"Tone and Voice" (2001) and *Sweet Anticipation* (2006); Mark Ellis, *A Chord in Time: The Evolution of the Augmented Sixth from Monteverdi to Mahler* (Ashgate, 2010); Herlinger (1981, 1985); Donahue, Thickstun & Liang, "Melody Transcription via Generative Pre-training" (ISMIR 2022) for the Hooktheory release.

## Appendix B — Corpora

| Corpus group | Works / movements used | Dating variable | Mode assignment | Editorial accidentals |
|---|---|---|---|---|
| Trecento (music21 corpus) | 103 | repertory c. 1325–1425 | final + signature | unmarked; used only for third-presence |
| Josquin Research Project (JRP) | 1,282 classifiable | composer birth cohorts | final + signature | marked; separable |
| CRIM (MEI) | 327 files, 259 works used | per-piece date_sort | used only for cadential-third measures | MEI accid vs accid.ges separable |
| Palestrina (music21 corpus) | 1,257 | c. 1554–94 | declared mode tags | unmarked |
| Tasso in Music Project | 503 files | per-work publication dates | final + signature | marked; separable |
| Corelli Opp. 1, 3, 4, 5, 6 (kern) | 250 movements | opus publication year, 1681–1714 | final + signature | unmarked |
| Vivaldi Opp. 1–2 (kern) | 87 movements | opus publication year, 1705/1709 | final + signature | unmarked |
| Bach 370 chorales (kern) | 370 | c. 1720–50 | kern key designation | unmarked |
| Scarlatti, Haydn, Mozart, Beethoven keyboard sonatas; Chopin preludes and mazurkas (kern) | 65 / 25 / 69 / 103 / 24 / 52 movements | standard dating | analyzed key (Appendix C) | unmarked |
| DCML Distant Listening Corpus | 1,326 movements (607 minor-mode used) | per-piece composed dates | expert-annotated key | n/a (annotations) |
| Hooktheory | 26,175 entries | c. 1960–2020, undated per song | declared by annotators | n/a |

Counts in this table are works or movements *used by the analyses*; corpus file counts differ where files were excluded or reclassified (trecento: 101 corpus files; Palestrina: 1,318 corpus files; JRP: 1,387 files of which 1,282 are mode-classifiable), and Appendix G's fingerprints are computed over files.

Scope restrictions, stated once. CRIM contributes only written-third measures, because CRIM mode inference disagrees with the ambit criterion in 38.1% of pieces (Appendix F). Palestrina contributes raised-degree measures under declared modes with editorial status unmarked, an exposure stated in Appendix E. Hooktheory contributes no augmented-sixth measure (spelling does not survive its sound-based encoding) and carries no per-song dates. The DCML corpus is a convenience sample of annotation projects, not a repertory survey; its bins pool genres.

## Appendix C — Operational definitions and statistical conventions

**Verticality; eligibility.** A verticality is a maximal time slice during which the sounding pitch set does not change; slice boundaries are the union of all note onsets and offsets. A verticality carrying at least two distinct sounding pitches is eligible, and eligible verticalities are the denominator for all augmented-sixth rates. One verticality contributes at most one event; a pair sustained across a slice boundary counts once, and an intervening single-pitch slice ends the sustain.

**Augmented-sixth detection.** A verticality containing a pitch pair ten semitones apart modulo twelve and five diatonic steps apart modulo seven — the spelling test that separates the augmented sixth from the minor seventh. Resolution: the next verticality contains both outer notes converged by semitone. Position: the verticality's start time over the piece's total duration; "terminal" means the final two percent.

**The three-chord test.** From a detected event: a dominant-quality sonority (the resolution pitch class with its major third) within four eligible verticalities; then a tonic arrival (the pitch class a fifth below the dominant, carrying a major or minor third) within eight more. Sensitivity at windows (2, 4) is reported in Appendix D and preserves every ordering claimed.

**Final sonority; ending classification.** The final sonority of a movement is the literal last slice — every pitch sounding at the end. No walk-back heuristic is applied: in repertoires whose movements end on bare octaves, any rule that walks back to the previous full sonority substitutes the (frequently major) penultimate chord for a genuinely open final and misclassifies the ending. Endings are classified by the third above the lowest final pitch: major, minor, or open. For keyboard and instrumental movements, destination (the minor/major identity of the movement itself) is classified by analyzed key with a tonic-rooted-final check, not by final-chord quality — final-chord quality conflates open endings with absent movements in exactly the repertoires where open endings are the norm. Vocal-corpus mode assignment: lowest final note against the key signature (with Palestrina's declared tags where present).

**Editorial policies.** Policy A counts an editorially supplied accidental as a raised degree, as published sources print it; policy B ignores editorial accidentals, reverting each note to its signature form. Every accidental-dependent series is reported under both (Appendix D, E); a file-exclusion policy (dropping files containing any editorial accidental) removes over 90 percent of relevant files and is unusable.

**Statistics.** All intervals are Wilson 95% confidence intervals for proportions. Cross-corpus comparisons are treated as snapshots, not series, wherever dating variables differ in kind (birth cohorts against publication years against composition dates).

**The modern raised-seventh measure.** In the Hooktheory corpus the raised-seventh rate is computed on melody lines: notes eleven semitones above the tonic over notes ten or eleven semitones above, in Aeolian-declared songs. The historical corpora count all voices at the seventh degree; the two measures are similar in construction but not identical in unit, and §6 compares their magnitudes with that disclosure.

**The cadence detector** recognises the two-voice sixth-to-octave clausula with stepwise contrary motion; its recall limits are reported in Appendix F and constrain every cadence-rate claim.

## Appendix D — Result tables

**Table D1 — The Picardy series (closed minor endings; fuller version of §6).** Trecento: 102/103 endings third-less. JRP birth cohorts: bare endings 93.5% (b. 1420–39) falling to 72.4% (b. 1440–59); among endings carrying a third, major 7/159 (4.4%) in b. 1440–59. Palestrina 574/580 (99.0%). Tasso 128/128, 38/39, 15/15. CRIM (minor-destination, third-bearing): 0/2 (1500–34), 3/6

(1535–54), 51/59 (1555–74), 17/17 (1575–1609). Corelli: Op. 1 9/9, Op. 3 7/7, Op. 4 6/8, Op. 5 1/2, Op. 6 14/15; Vivaldi Op. 1 14/16, Op. 2 0 closed (27 open); pooled 51/57 = 89.5% [78.9–95.1], open finals 117/174 minor movements. Bach chorales 130/138 = 94.2% [89.0–97.0]. DCML bins (minor-mode movements; Picardy / minor third / open / other): 1600–24: 10/1/0/2; 1625–49: 56/2/21/2; 1650–74: 1/0/0/0; 1675–99: 1/0/43/33 (closed n=1); 1700–24: 7/35/56/15; 1725–49: 6/11/35/11; 1750–74: 0/14/14/3; 1775–99: 3/28/7/4; 1800–24: 3/6/4/3; 1825–49: 11/30/12/18; 1850–74: 1/5/4/2; 1875–99: 3/14/5/11; 1900–24: 1/9/11/11; 1925–49: 0/10/7/5. Scarlatti 1/4 closed (16/20 minor endings open); Haydn+Mozart 1/8; Beethoven 6/19; Chopin 3/21 (roughly a third open). Hooktheory 366/2,778 = 13.2% [12.0–14.5].

**Table D2 — Raised-sixth conditional cells, policy A (accidentals as printed), window ±2 notes, same voice.** As printed in §11. Window sensitivity: at ±1 the ASC-near cells read 99.5 / 99.3 / 95.7 / 99.5 (JRP / Palestrina / Tasso / Bach); at ±3, 65.0 / 79.3 / 68.9 / 79.7. What holds in all four corpora at every window is the within-direction proximity multiplier; the cross-comparison — descending-near exceeding ascending-far — is decisive in Palestrina at every window and in Tasso at ±2, does not hold in the Bach chorales (28.8% against 33.1% at ±2; 19.5% against 31.8% at ±3), and is unevaluable in JRP, whose descending-near cells are too small (n = 20 at ±2).

**Table D3 — Raised-sixth conditional cells, policy B (editorial accidentals reverted), window ±2.** JRP: ASC-near 74.1% (20/27) [55.3–86.8], ASC-far 0.5% (n=15,347), DESC-far 0.2% (n=29,916). Tasso: ASC-near 56.1% (64/114) [47.0–64.9], ASC-far 18.4% (n=2,701), DESC-near 36.1% (n=61), DESC-far 3.7% (n=6,216). Palestrina and the Bach chorales carry no editorial marking in these encodings (0 marked notes in 153,963 and 41,994 respectively), so policy B equals policy A there by construction and those corpora cannot participate in the contrast; the decisive descending-near cell is Palestrina's and carries that exposure (Appendix E).

**Table D4 — Dated conditional series (ASC-near cell).** JRP by birth cohort, policy A: 89.7% (26/29, b. 1420–39), 87.5% (140/160, b. 1440–59), 93.8% (15/16, b. 1460–79); policy B: 68.4% (13/19) in b. 1440–59, other cohorts empty of scribal contexts. Tasso by period, policy A: 67.9% (19/28), 84.0% (68/81), 78.9% (15/19); policy B: 26.3% (5/19) [11.8–48.8], 61.0% (47/77) [49.9–71.2], 66.7% (12/18) [43.7–83.7].

**Table D5 — Raised-degree marginals under editorial policies (per-cohort rates, policy A / policy B).** Raised seventh, JRP: 8.4/0.0 (19/226, b. 1390–1419), 8.1/0.4 (842/10,413), 5.2/0.3 (4,378/84,833), 5.8/0.3 (393/6,757); Tasso: 12.3/9.3 (1,195/9,711), 15.3/12.6 (1,588/10,408), 12.9/11.1 (879/6,805). Raised sixth, JRP: 0.5/0.0, 1.1/0.3, 1.1/0.1, 1.7/0.0; Tasso: 4.1/3.1, 7.7/6.2, 3.2/2.9. In the Josquin corpus 95.5% of minor-destination pieces contain at least one editorial accidental; in Tasso, 91.4%.

**Table D6 — The augmented-sixth series (events / eligible verticalities; rate per 1,000).** Renaissance pooled 25/1,128,193 = 0.022 (JRP 17/660,176; Tasso 6/133,827; trecento 2/38,598; Palestrina 0/295,592); adjudication: 18 of 25 editorial, 5 resolving, 0 terminal, exactly one scribal-and-resolving (mid-piece). Bach chorales 3/30,067 = 0.100 (all scribal, all resolving, all completing the three-chord chain, none terminal). Scarlatti 11/38,073 = 0.289; Haydn 39/18,687 = 2.087; Mozart 148/48,617 = 3.044; Beethoven 407/124,414 = 3.271; Chopin preludes 28/8,663 = 3.232; Chopin mazurkas 112/21,092 = 5.310. Three-chord chains (windows 4/8): Scarlatti 6/11 (54.5%), Haydn 15/39 (38.5%), Mozart 43/148 (29.1%), Beethoven 78/407 (19.2%), preludes 7/28, mazurkas 36/112; terminal chains 3 of 745 keyboard events; windows 2/4 preserve the ordering (12.8 / 9.5 / 6.9% for Haydn / Mozart / Beethoven). Destination by analyzed key (minor-key rate / major-key rate per 1,000): Scarlatti 0.597 (7/11,732) / 0.174 (4/22,960); Haydn 3.783 (14/3,701) / 1.668 (25/14,986); Mozart 7.785 (29/3,725) / 2.585 (108/41,782); Beethoven 2.958 (87/29,409) / 3.168 (253/79,871); Chopin preludes 3.754 (17/4,529) / 0.304 (1/3,293); mazurkas 5.632 (46/8,167) / 4.596 (40/8,704); movements failing the

tonic-rooted key check are excluded and reported in the analysis output. Beethoven decile distribution: 47, 37, 16, 30, 74, 36, 45, 36, 52, 34; 1/407 in the final two percent.

**Table D7 — The mode field.** Lydian 8.0% → 2.9% → 0.0% across JRP cohorts, 1.2% in Tasso; Locrian 5/1,282 = 0.39% (JRP); Phrygian 14.1% sacred vs 2.9% secular; destination balance 38.6/61.0 (JRP) vs 39.0/60.6 (Tasso); Ionian's share of finals 19.2% vs 18.9%. Hooktheory: Ionian 50.4, Aeolian 39.3, Mixolydian 3.9, Dorian 3.8, Phrygian 1.1, Lydian 1.0, Locrian 0.3, other 0.3 (percent of 26,175). Cadence repair at finals: Ionian 63.9% repair-free vs Aeolian 8.5%; the repair-free rate aggregated over all cadences runs roughly 19–28% across cohorts with no trend. JRP mode-distribution highlight: Dorian is the most common mode of the classifiable repertory at 37.2% of works.

**Table D8 — "Natural minor": dated attestations and frequency.** Verified attestations: Goodban 1820 (p. 32), Relfe 1821 (p. 32), Busby 1828, Mason 1854 (§72, "natural" vs "artificial"), Root 1855 (§72), *Proceedings of the Musical Association* 1874–75 (p. 76), Stainer & Barrett 1876 (s.v. "A," as secondary name). Standard harmony texts through 1905 (Marx trans. 1854, Weber trans. 1846, Richter trans. 1867, Stainer 1871, Macfarren 1880, Prout 1889, Goetschius 1889 and 1892, Jadassohn trans. 1893, York 1901, Foote & Spalding 1905) use "Aeolian," "descending/melodic form," or "normal minor," never "natural minor." Google Books Ngram (English): flat low baseline 1850s–1920s, sustained climb from the 1930s, takeoff after 1960, plateau 2000s–2010s.

**Table D9 — The curriculum record (documentary).** Chorales as teaching material, dated: J. S. Bach's own classroom (C. P. E. Bach's 1775 testimony: pupils write alto and tenor to given outer voices); Kirnberger 1771 (I:157 — the chorales as "best models," the completion exercise prescribed); Becker 1832 (69 chorales packaged with figured bass as exercises); Marx 1837 (Bach settings compared as models); Erk 1850 (preface protesting the chorales' use as "ordinary exercise-pieces for thoroughbass students"); Richter 1853 (Leipzig Conservatory text; chorale harmonization central, models unattributed); Goetschius 1882 (Stuttgart; St. Matthew Passion chorales assigned as models); Chadwick 1897 (New England Conservatory; 17 Bach chorales appended); Prout 1901–03 (chorale-harmonization chapter; "very nearly four-hundred" chorales cited as the norm); Riemenschneider 1941. Negative controls: Cherubini, Day, Macfarren, Jadassohn, Riemann (no chorale models); Paris Conservatoire (chorale harmonization only in d'Indy's rejected 1892 Projet). Picardy third as taught topic, dated: Rousseau 1768 (lexicography of an abandoned practice); Crotch 1812 (major ending prescribed as the general rule; name as historical footnote); Busby 1828 (antiquarian dictionary entry); Ouseley 1868 (cadences chapter); Prout 1889 ("still not infrequently employed, especially in church music"); Foote & Spalding 1905/1936 (historicized); Kitson 1914 (assumed knowledge); Hull 1915 (modal-revival resource); German lineage 1846–1896 (term never taught). Institutional frame: Royal Academy of Music prospectuses, 151 of 152 digitized items 1880–2020 — harmony and counterpoint constant subjects, the era's textbook authors (Prout, Macpherson, Stainer, Macfarren) on staff, no set texts named in the genre; earliest scannable chorale-examination language 1992.

## Appendix E — Editorial provenance

Every accidental-dependent measurement is exposed to a question the sources cannot settle by themselves: whether an accidental was written by a scribe or supplied by a modern editor. The Josquin Research Project and the Tasso corpus mark the distinction, and every such series is reported under policy A (as printed) and policy B (editorial accidentals reverted to signature form) in Appendix D. What the policies show: in the Josquin corpus the raised seventh is almost entirely editorial — policy B collapses the fifteenth-century rate from 5.2–8.4 percent to 0.0–0.4 percent, so roughly 95 percent of the raised sevenths in the printed figure were supplied by modern editors — while in the Tasso corpus the same removal barely moves the rate, so Tasso's raised sevenths are genuinely notated. The notated rise across the period is therefore roughly thirty-fold; how much of it is musical practice and how much

notational practice, the corpora cannot say. Palestrina's encoding declares modes but carries no editorial marking on accidentals: a declared mode does not identify which accidentals are editorial, so Palestrina's raised-degree cells carry an unquantified editorial exposure, stated here as a limitation, and no raised-degree conclusion rests on Palestrina alone. The augmented-second result of §11 is robust in the conservative direction regardless: editors supplying ficta tend to create augmented seconds rather than suppress them, and the observed count is near zero under any policy. For the Renaissance augmented-sixth events, provenance is decisive and event-level: 18 of the 25 involve an editorially supplied accidental (Appendix D, Table D6).

## Appendix F — Detector and classifier reliability

**The cadence detector.** No independent expert annotation has been obtained; sensitivity analysis over the detector's own criteria can establish robustness and detect systematic misses, but cannot establish precision against an external standard. Robustness to the arrival-duration threshold is good (repair-free fractions move by 0.1–1.0 points across settings). The systematic miss is real and period-dependent: the detector recognises only the sixth-to-octave clausula and is blind to the authentic cadence with a leaping bass, and that blindness worsens over time — the missed share runs about 20 percent in the Josquin corpus and 43–58 percent across the Tasso periods. Every between-corpus cadence-rate comparison in this paper is read with that recalibration.

**Mode and key classifiers.** Mode-from-final misreads pieces ending on a bare fifth or with a crossing bass. CRIM mode inference disagrees with the traditional ambit criterion in 38.1 percent of pieces and assigns seven mass movements ending on B to Locrian — a diagnosable classifier failure, which is why CRIM contributes only written-third measures. For instrumental corpora, movement destination is classified by analyzed key with a tonic-rooted-final gate (Appendix C); movements failing the gate are excluded rather than guessed. Final-chord-quality classification of destination is not used, because it cannot see the minor identity of movements that end open — a systematic property of the very repertoires (Scarlatti above all, sixteen of twenty minor endings open) where the question matters most.

**Parsers.** The kern parser used for the vocal corpora excludes divided staves — 105 JRP files, recovered in full by a repaired parser whose note-level output matches an independent music21 parse exactly (159,471 notes); including the recovered files moves no mode-field share by more than half a percentage point, though the exclusion is non-random by composer. Keyboard and instrumental kern, whose chord tokens and divided staves the simple parser cannot represent, is parsed through music21 throughout.

## Appendix G — Reproduction

The analysis code, the per-driver manifests, the corpus fingerprints, and a single command that regenerates and checks every headline value accompany this paper as an analysis bundle. All corpora are public: the Josquin Research Project, Tasso in Music Project, CRIM, and the kern editions of the trecento, Palestrina, chorale, Corelli, Vivaldi, and keyboard repertoires are downloadable from their project sites; the DCML Distant Listening Corpus and Hooktheory release are published datasets. As of 2 August 2026 the full check suite — eleven headline values spanning the augmented-second counts, the Picardy cells, the augmented-sixth series, the editorial-share figures, the CRIM series, and the parser-recovery test — passes end to end on freshly downloaded corpora. Corpus fingerprints are computed over sorted per-file checksums; the CRIM corpus is served live by its project and drifts (5 of 327 files differ from the fingerprinted state without changing any cell reported here), which is stated so that future reruns expect it.