Jazz Scales Mapped to Diatonic Modes, Pentatonic Modes and the Rules of "Ionian-ization"

A table of the scales that jazz teaching names, each explained in MeloPresto™ terms

1. Result

Of the 67 scales listed below, 60 are explained and 7 are not.

Class	Explanation	Scales
0	the chromatic scale	2
1	a mode of the diatonic scale	7
2	a mode of the pentatonic scale	5
3	five notes of a diatonic mode, or of a minor mode with rules applied	6
4	a mode plus one or two added notes	10
5	a minor mode with rules of "Ionian-ization" applied	10
6	another mode of a class-5 scale	20
7	repeats inside the octave of 12-edo: not explained	5
8	not explained	2
	total	67

Classes 0, 1 and 2 are the three well-formed scales (section 5): the chromatic scale, and the modes of the diatonic scale and of the pentatonic scale. They are 14 of the 67 scales. Classes 3 and 4 add 16 scales, each a part of a mode or a mode with one or two notes added; 2 of the 16 need a rule as well. Classes 5 and 6 include 30 scales: each is one of the three minor modes (Re-mode, La-mode, Mi-mode) with one or more of the rules of "Ionian-ization" applied, taken as it stands (class 5) or in another of its modes (class 6).

The 5 scales of class 7 and the 2 scales of class 8 are the scales that neither the diatonic scale, nor the pentatonic scale, nor the rules produce (section 6).

2. Terms

- **"Ionian-ization"**: the rebuilding of a minor mode's final cadence toward Do-mode's final cadence (*A Cadence of Change*; *Minor Scales in MeloPresto*, section 4).
- **Rule 1, the Dominant-ization**: the seventh degree raised. In La-mode, Si in place of So.
- **Rule 2a**: the sixth degree raised directly beside the raised seventh, which removes the A2 (augmented second). In La-mode, Fi in place of Fa. Rule 2a applies only in La-mode and Mi-mode, whose sixth degree lies an m2 above the fifth degree. Rule 2, "Avoiding the A2", has three parts in *Minor Scales in MeloPresto*, and Rule 2a is the only part that changes a note: Rule 2b allows either sixth when one note stands between the sixth degree and the raised seventh, and Rule 2c keeps the seventh unraised under tonic or subdominant harmony.
- **Rule 3, the Major-ization**: an M3 above the tonic. In La-mode, Di in place of Do.
- **Rule 4, the Fortification**: the A6 (augmented sixth) above the sixth degree. In La-mode, Ri. A seven-note scale has one note per degree, so the table writes Ri in place of Re.

- **Minor modes**: Re-mode, La-mode and Mi-mode, the three diatonic modes whose tonic triads are minor.
- **Pentatonic scale**: the five stack-of-fifths positions Do, So, Re, La, Mi (`Theory.md`, section 5.4). Its modes are named in the form `[scale name] [tonic]-mode`: pentatonic Do-mode, pentatonic La-mode, and so on (`Nomenclature.md`, section 3).
- **TiMS** (the West's Traditional incoherent Music System): conventional Western notation and theory. The jazz names in the first column are TiMS names, written in quotation marks because the MeloPresto Music System does not use them (`Nomenclature.md`, section 3); they are the things being mapped.
- **12-edo**: the tuning that divides P8 into twelve equal steps; *edo* stands for *equal division of the octave*.
- **Degrees column**: each note as jazz teaching spells it against the major scale of the same tonic (♭ lowers by A1, ♯ raises by A1).

Three sets that the rules produce have TiMS names, and the table uses those names for the rotations of each set:

TiMS name	Rules	Notes
harmonic minor	La-mode + Rule 1	La Ti Do Re Mi Fa Si
melodic minor	La-mode + Rules 1, 2a	La Ti Do Re Mi Fi Si
harmonic major	La-mode + Rules 1, 3	La Ti Di Re Mi Fa Si

3. Method

Each scale is written as a set of stack-of-fifths positions, so that a spelled note (Si) and its 12-edo neighbour (Le) stay different notes. The script `jazz_scales_mapped.py`, beside this file, tests each set in this order and stops at the first test that succeeds.

1. The set is seven consecutive stack-of-fifths positions: a mode of the diatonic scale, named by the position of the tonic in the run.
2. The set is five consecutive stack-of-fifths positions: a mode of the pentatonic scale.
3. The set repeats inside the octave of 12-edo (class 7).
4. The set equals a minor mode with a choice of Rules 1, 2a, 3 and 4 applied (class 5), or is another mode of such a set (class 6). Among several derivations the script prefers the same tonic, then the fewest rules, then La-mode.
5. Removing one or two notes leaves a diatonic mode, a pentatonic mode or a class-5 set (class 4).
6. The set has fewer than seven notes and lies inside a diatonic mode or a class-5 set (class 3).

The rules explain many sets, so a match needs a base rate. There are 155 seven-note spellings with one note per degree, each degree flat, natural or sharp, that rise strictly inside the octave. Tests 1 and 4 explain 68 of them (44%). Tests 1 and 4 explain 37 of the 39 seven-note scales in this table (95%).

The last column of the seven-note tables is independent of the rules: the diatonic mode on the same tonic that differs from the scale in the fewest degrees.

4. The tables

Chromatic scale

Jazz name	Degrees	Class	Explanation	Notes
"Chromatic, the segment that includes Si"	1 ♯1 2 ♭3 3 4 ♯4 5 ♯5 6 ♭7 7	0	the chromatic scale: twelve consecutive stack-of-fifths positions	Do Di Re Me Mi Fa Fi So Si La Te Ti
"Chromatic, the segment that includes Le"	1 ♯1 2 ♭3 3 4 ♯4 5 ♭6 6 ♭7 7	0	the chromatic scale: twelve consecutive stack-of-fifths positions	Do Di Re Me Mi Fa Fi So Le La Te Ti

Diatonic modes

Jazz name	Degrees	Class	Explanation	Notes
"Ionian (major)"	1 2 3 4 5 6 7	1	Do-mode	Do Re Mi Fa So La Ti
"Dorian"	1 2 ♭3 4 5 6 ♭7	1	Re-mode	Re Mi Fa So La Ti Do
"Phrygian"	1 ♭2 ♭3 4 5 ♭6 ♭7	1	Mi-mode	Mi Fa So La Ti Do Re
"Lydian"	1 2 3 ♯4 5 6 7	1	Fa-mode	Fa So La Ti Do Re Mi
"Mixolydian"	1 2 3 4 5 6 ♭7	1	So-mode	So La Ti Do Re Mi Fa
"Aeolian (natural minor)"	1 2 ♭3 4 5 ♭6 ♭7	1	La-mode	La Ti Do Re Mi Fa So
"Locrian"	1 ♭2 ♭3 4 ♭5 ♭6 ♭7	1	Ti-mode	Ti Do Re Mi Fa So La

Melodic minor and its modes

Jazz name	Degrees	Class	Explanation	Notes	Nearest diatonic mode on the same tonic
"Melodic minor ascending (jazz minor)"	1 2 ♭3 4 5 6 7	5	La-mode + Rules 1, 2a (TiMS: melodic minor)	La Ti Do Re Mi Fi Si	Re-mode, 7th raised; Do-mode, 3rd lowered
"Dorian ♭2 (Phrygian ♮6)"	1 ♭2 ♭3 4 5 6 ♭7	6	melodic minor Ti-mode	Ti Do Re Mi Fi Si La	Mi-mode, 6th raised; Re-mode, 2nd lowered
"Lydian augmented"	1 2 3 ♯4 ♯5 6 7	6	melodic minor Do-mode	Do Re Mi Fi Si La Ti	Fa-mode, 5th raised
"Lydian dominant (acoustic, overtone)"	1 2 3 ♯4 5 6 ♭7	6	melodic minor Re-mode	Re Mi Fi Si La Ti Do	So-mode, 4th raised; Fa-mode, 7th lowered
"Mixolydian ♭6 (Hindu, Aeolian dominant)"	1 2 3 4 5 ♭6 ♭7	5	La-mode + Rule 3	La Ti Di Re Mi Fa So	La-mode, 3rd raised; So-mode, 6th lowered
"Locrian ♯2 (half-diminished)"	1 2 ♭3 4 ♭5 ♭6 ♭7	6	melodic minor Fi-mode	Fi Si La Ti Do Re Mi	Ti-mode, 2nd raised; La-mode, 5th lowered
"Altered (super-Locrian, diminished whole-tone)"	1 ♭2 ♭3 ♭4 ♭5 ♭6 ♭7	6	melodic minor Si-mode	Si La Ti Do Re Mi Fi	Ti-mode, 4th lowered

Harmonic minor and its modes

Jazz name	Degrees	Class	Explanation	Notes	Nearest diatonic mode on the same tonic
"Harmonic minor"	1 2 ♭3 4 5 ♭6 7	5	La-mode + Rule 1 (TiMS: harmonic minor)	La Ti Do Re Mi Fa Si	La-mode, 7th raised

Jazz name	Degrees	Class	Explanation	Notes	Nearest diatonic mode on the same tonic
"Locrian ♮6"	1 ♭2 ♭3 4 ♭5 6 ♭7	6	harmonic minor Ti-mode	Ti Do Re Mi Fa Si La	Ti-mode, 6th raised
"Ionian ♯5 (Ionian augmented)"	1 2 3 4 ♯5 6 7	6	harmonic minor Do-mode	Do Re Mi Fa Si La Ti	Do-mode, 5th raised
"Dorian ♯4 (Ukrainian Dorian, Romanian minor)"	1 2 ♭3 ♯4 5 6 ♭7	6	harmonic minor Re-mode	Re Mi Fa Si La Ti Do	Re-mode, 4th raised
"Spanish or Jewish scale (fifth mode of harmonic minor)"	1 ♭2 3 4 5 ♭6 ♭7	5	Mi-mode + Rule 3	Mi Fa Si La Ti Do Re	Mi-mode, 3rd raised
"Lydian ♯2 (Lydian ♯9)"	1 ♯2 3 ♯4 5 6 7	6	harmonic minor Fa-mode	Fa Si La Ti Do Re Mi	Fa-mode, 2nd raised
"Ultralocrian (altered diminished)"	1 ♭2 ♭3 ♭4 ♭5 ♭6 𝄫7	6	harmonic minor Si-mode	Si La Ti Do Re Mi Fa	Ti-mode, 4th lowered, 7th lowered

Harmonic major and its modes

Jazz name	Degrees	Class	Explanation	Notes	Nearest diatonic mode on the same tonic
"Harmonic major"	1 2 3 4 5 ♭6 7	5	La-mode + Rules 1, 3 (TiMS: harmonic major)	La Ti Di Re Mi Fa Si	Do-mode, 6th lowered
"Dorian ♭5"	1 2 ♭3 4 ♭5 6 ♭7	6	harmonic major Ti-mode	Ti Di Re Mi Fa Si La	Re-mode, 5th lowered
"Phrygian ♭4"	1 ♭2 ♭3 ♭4 5 ♭6 ♭7	6	harmonic major Di-mode	Di Re Mi Fa Si La Ti	Mi-mode, 4th lowered
"Lydian ♭3 (Lydian diminished)"	1 2 ♭3 ♯4 5 6 7	6	harmonic major Re-mode	Re Mi Fa Si La Ti Di	Fa-mode, 3rd lowered
"Mixolydian ♭2 (Mixolydian ♭9)"	1 ♭2 3 4 5 6 ♭7	6	harmonic major Mi-mode	Mi Fa Si La Ti Di Re	So-mode, 2nd lowered
"Lydian augmented ♯2"	1 ♯2 3 ♯4 ♯5 6 7	6	harmonic major Fa-mode	Fa Si La Ti Di Re Mi	Fa-mode, 2nd raised, 5th raised
"Locrian 𝄫7"	1 ♭2 ♭3 4 ♭5 ♭6 𝄫7	6	harmonic major Si-mode	Si La Ti Di Re Mi Fa	Ti-mode, 7th lowered

Scales with two A2 steps

Jazz name	Degrees	Class	Explanation	Notes	Nearest diatonic mode on the same tonic
"Double harmonic major (Byzantine)"	1 ♭2 3 4 5 ♭6 7	5	Mi-mode + Rules 1, 3	Mi Fa Si La Ti Do Ri	Mi-mode, 3rd raised, 7th raised; Do-mode, 2nd lowered, 6th lowered
"Hungarian minor"	1 2 ♭3 ♯4 5 ♭6 7	5	La-mode + Rules 1, 4	La Ti Do Ri Mi Fa Si	La-mode, 4th raised, 7th raised; Fa-mode, 3rd lowered, 6th lowered
"Oriental (fifth mode of double harmonic major)"	1 ♭2 3 4 ♭5 6 ♭7	6	Ti-mode of (La-mode + Rules 1, 4)	Ti Do Ri Mi Fa Si La	Ti-mode, 3rd raised, 6th raised; So-mode, 2nd lowered, 5th lowered

Other seven-note scales found in jazz scale books

Jazz name	Degrees	Class	Explanation	Notes	Nearest diatonic mode on the same tonic
"Neapolitan minor"	1 ♭2 ♭3 4 5 ♭6 7	5	Mi-mode + Rule 1	Mi Fa So La Ti Do Ri	Mi-mode, 7th raised
"Neapolitan major"	1 ♭2 ♭3 4 5 6 7	5	Mi-mode + Rules 1, 2a	Mi Fa So La Ti Di Ri	Mi-mode, 6th raised, 7th raised; Re-mode, 2nd lowered, 7th raised; Do-mode, 2nd lowered, 3rd lowered
"Hungarian major"	1 ♯2 3 ♯4 5 6 ♭7	8	not explained	—	So-mode, 2nd raised, 4th raised; Fa-mode, 2nd raised, 7th lowered
"Major Locrian (Arabian)"	1 2 3 4 ♭5 ♭6 ♭7	6	Ti-mode of (Mi-mode + Rules 1, 2a)	Ti Di Ri Mi Fa So La	Ti-mode, 2nd raised, 3rd raised; La-mode, 3rd raised, 5th lowered; So-mode, 5th lowered, 6th lowered
"Lydian minor"	1 2 3 ♯4 5 ♭6 ♭7	5	La-mode + Rules 3, 4	La Ti Di Ri Mi Fa So	La-mode, 3rd raised, 4th raised; So-mode, 4th raised, 6th lowered; Fa-mode, 6th lowered, 7th lowered
"Leading whole-tone"	1 2 3 ♯4 ♯5 ♯6 7	6	Fa-mode of (Mi-mode + Rules 1, 2a)	Fa So La Ti Di Ri Mi	Fa-mode, 5th raised, 6th raised
"Persian"	1 ♭2 3 4 ♭5 ♭6 7	6	Ti-mode of (Mi-mode + Rules 1, 4)	Ti Do Ri Mi Fa So Li	Ti-mode, 3rd raised, 7th raised
"Enigmatic"	1 ♭2 3 ♯4 ♯5 ♯6 7	8	not explained	—	Fa-mode, 2nd lowered, 5th raised, 6th raised

Bebop scales

Jazz name	Degrees	Class	Explanation	Notes
"Bebop dominant"	1 2 3 4 5 6 ♭7 7	4	So-mode + Fi OR Do-mode + Te	So La Ti Do Re Mi Fa Fi
"Bebop major (major sixth diminished)"	1 2 3 4 5 ♯5 6 7	4	Do-mode + Si	Do Re Mi Fa So Si La Ti
"Bebop Dorian (bebop minor)"	1 2 ♭3 3 4 5 6 ♭7	4	Re-mode + Fi OR So-mode + Te	Re Mi Fa Fi So La Ti Do
"Bebop melodic minor (minor sixth diminished)"	1 2 ♭3 4 5 ♭6 6 7	4	La-mode + Rule 1, keeping Fi OR La-mode + Rules 1, 2a, keeping Fa	La Ti Do Re Mi Fa Fi Si
"Bebop harmonic minor"	1 2 ♭3 4 5 ♭6 ♭7 7	4	La-mode + Si	La Ti Do Re Mi Fa So Si
"Bebop Locrian (half-diminished bebop)"	1 ♭2 ♭3 4 ♭5 5 ♭6 ♭7	4	Ti-mode + Fi OR Mi-mode + Te	Ti Do Re Mi Fa Fi So La

Pentatonic scales

Jazz name	Degrees	Class	Explanation	Notes
"Major pentatonic"	1 2 3 5 6	2	pentatonic Do-mode	Do Re Mi So La
"Minor pentatonic"	1 ♭3 4 5 ♭7	2	pentatonic La-mode	La Do Re Mi So
"Suspended (Egyptian) pentatonic"	1 2 4 5 ♭7	2	pentatonic Re-mode	Re Mi So La Do
"Pentatonic mode 3 (Man Gong, blues minor)"	1 ♭3 4 ♭6 ♭7	2	pentatonic Mi-mode	Mi So La Do Re

Jazz name	Degrees	Class	Explanation	Notes
"Pentatonic mode 5 (Ritsu, blues major)"	1 2 4 5 6	2	pentatonic So-mode	So La Do Re Mi
"Minor sixth pentatonic"	1 ♭3 4 5 6	3	5 notes of Re-mode	Re Fa So La Ti
"Dominant pentatonic"	1 2 3 5 ♭7	3	5 notes of So-mode	So La Ti Re Fa
"Kumoi"	1 2 ♭3 5 6	3	5 notes of Re-mode	Re Mi Fa La Ti
"In-sen"	1 ♭2 4 5 ♭7	3	5 notes of Mi-mode	Mi Fa La Ti Re
"Hirajoshi"	1 2 ♭3 5 ♭6	3	5 notes of La-mode	La Ti Do Mi Fa
"Major ♭6 pentatonic"	1 2 3 5 ♭6	3	5 notes of (La-mode + Rule 3)	La Ti Di Mi Fa

Blues scales

Jazz name	Degrees	Class	Explanation	Notes
"Blues scale, spelled with ♯4"	1 ♭3 4 ♯4 5 ♭7	4	pentatonic La-mode + Ri	La Do Re Ri Mi So
"Blues scale, spelled with ♭5"	1 ♭3 4 ♭5 5 ♭7	4	pentatonic La-mode + Me	La Do Re Me Mi So
"Major blues"	1 2 ♯2 3 5 6	4	pentatonic Do-mode + Ri	Do Re Ri Mi So La
"Nine-note blues scale"	1 2 ♭3 3 4 5 6 ♭7 7	4	So-mode + Te, Fi OR Re-mode + Fi, Di OR Do-mode + Me, Te	So La Te Ti Do Re Mi Fa Fi

Scales that repeat inside the octave of 12-edo

Jazz name	Degrees	Class	Explanation	Notes
"Whole-tone"	1 2 3 ♯4 ♯5 ♭7	7	repeats 6 times inside the octave of 12-edo	—
"Diminished, whole step first"	1 2 ♭3 4 ♭5 ♭6 6 7	7	repeats 4 times inside the octave of 12-edo	—
"Diminished, half step first"	1 ♭2 ♭3 3 ♯4 5 6 ♭7	7	repeats 4 times inside the octave of 12-edo	—
"Augmented (hexatonic)"	1 ♯2 3 5 ♭6 7	7	repeats 3 times inside the octave of 12-edo	—
"Tritone scale"	1 ♭2 3 ♭5 5 ♭7	7	repeats 2 times inside the octave of 12-edo	—

5. The list as cycles

A mode is a rotation of a scale's cycle, so all the modes of one cycle are one scale. Counted that way, the 67 names are 26 cycles.

Kind of cycle	Cycles	Names
well-formed scale	3	14
minor mode with rules applied	7	30
mode plus added notes	6	10
part of a mode	4	6
repeats inside the octave of 12-edo	4	5
not explained	2	2
total	26	67

A **well-formed scale** is a run of consecutive stack-of-fifths positions with at most two step sizes (`Theory.md`, section 5). Each cycle below is written from the tonic of its first name.

Kind	Notes	Cycle	Names
well-formed scale	12	A1 m2 m2 A1 m2 A1 m2 A1 m2 m2 A1 m2	"Chromatic, the segment that includes Si"; "Chromatic, the segment that includes Le"
well-formed scale	7	M2 M2 m2 M2 M2 M2 m2	"Ionian (major)"; "Dorian"; "Phrygian"; "Lydian"; "Mixolydian"; "Aeolian (natural minor)"; "Locrian"
well-formed scale	5	M2 M2 m3 M2 m3	"Major pentatonic"; "Minor pentatonic"; "Suspended (Egyptian) pentatonic"; "Pentatonic mode 3 (Man Gong, blues minor)"; "Pentatonic mode 5 (Ritsu, blues major)"
minor mode with rules applied	7	M2 m2 M2 M2 M2 M2 m2	"Melodic minor ascending (jazz minor)"; "Dorian ♭2 (Phrygian ♮6)"; "Lydian augmented"; "Lydian dominant (acoustic, overtone)"; "Mixolydian ♭6 (Hindu, Aeolian dominant)"; "Locrian ♯2 (half-diminished)"; "Altered (super-Locrian, diminished whole-tone)"
minor mode with rules applied	7	M2 m2 M2 M2 m2 A2 m2	"Harmonic minor"; "Locrian ♮6"; "Ionian ♯5 (Ionian augmented)"; "Dorian ♯4 (Ukrainian Dorian, Romanian minor)"; "Spanish or Jewish scale (fifth mode of harmonic minor)"; "Lydian ♯2 (Lydian ♯9)"; "Ultralocrian (altered diminished)"
minor mode with rules applied	7	M2 M2 m2 M2 m2 A2 m2	"Harmonic major"; "Dorian ♭5"; "Phrygian ♭4"; "Lydian ♭3 (Lydian diminished)"; "Mixolydian ♭2 (Mixolydian ♭9)"; "Lydian augmented ♯2"; "Locrian 𝄫7"
minor mode with rules applied	7	m2 A2 m2 M2 m2 A2 m2	"Double harmonic major (Byzantine)"; "Hungarian minor"; "Oriental (fifth mode of double harmonic major)"
minor mode with rules applied	7	m2 M2 M2 M2 m2 A2 m2	"Neapolitan minor"
minor mode with rules applied	7	m2 M2 M2 M2 M2 M2 m2	"Neapolitan major"; "Major Locrian (Arabian)"; "Lydian minor"; "Leading whole-tone"
minor mode with rules applied	7	m2 A2 m2 m2 M2 A2 m2	"Persian"
mode plus added notes	8	M2 M2 m2 M2 M2 m2 A1 m2	"Bebop dominant"; "Bebop Dorian (bebop minor)"; "Bebop Locrian (half-diminished bebop)"
mode plus added notes	8	M2 M2 m2 M2 A1 m2 M2 m2	"Bebop major (major sixth diminished)"; "Bebop harmonic minor"
mode plus added notes	8	M2 m2 M2 M2 m2 A1 M2 m2	"Bebop melodic minor (minor sixth diminished)"
mode plus added notes	6	m3 M2 A1 m2 m3 M2	"Blues scale, spelled with ♯4"; "Major blues"
mode plus added notes	6	m3 M2 m2 A1 m3 M2	"Blues scale, spelled with ♭5"
mode plus added notes	9	M2 m2 A1 m2 M2 M2 m2 A1 m2	"Nine-note blues scale"
part of a mode	5	m3 M2 M2 M2 m3	"Minor sixth pentatonic"; "Dominant pentatonic"

Kind	Notes	Cycle	Names
part of a mode	5	M2 m2 M3 M2 m3	"Kumoi"; "In-sen"
part of a mode	5	M2 m2 M3 m2 M3	"Hirajoshi"
part of a mode	5	M2 M2 m3 m2 M3	"Major ♭6 pentatonic"
repeats inside the octave of 12-edo	6	M2 M2 M2 M2 d3 M2	"Whole-tone"
repeats inside the octave of 12-edo	8	M2 m2 M2 m2 M2 A1 M2 m2	"Diminished, whole step first"; "Diminished, half step first"
repeats inside the octave of 12-edo	6	A2 m2 m3 m2 A2 m2	"Augmented (hexatonic)"
repeats inside the octave of 12-edo	6	m2 A2 d3 A1 m3 M2	"Tritone scale"
not explained	7	A2 m2 M2 m2 M2 m2 M2	"Hungarian major"
not explained	7	m2 A2 M2 M2 M2 m2 m2	"Enigmatic"

How many seven-note cycles are possible. With one note per degree, each degree flat, natural or sharp, 46 seven-note cycles exist. Jazz teaching names 10 of them. The rules explain 12 of the 46, and 8 of the 10 that jazz names. Only 3 cycles use M2 and m2 steps alone (five M2 and two m2 is the only count that sums to P8): the diatonic scale, melodic minor and "Neapolitan major". Jazz names all 3.

The well-formed cardinalities. At each of twelve generators tested between 686 and 720 cents, a stack of fifths has at most two step sizes at 2, 3, 5, 7 and 12 notes. The next such count is 19 notes at every tested generator below 700 cents and 17 notes at every tested generator above 700 cents. A well-formed scale is a segment of the stack of fifths centred on Re, and a twelve-note segment cannot be centred. The chromatic scale therefore has two variants: the segment from Me to Si, which includes Si, and the segment from Le to Di, which includes Le. Together they are the thirteen-note segment from Le to Si, centred on Re. Both variants have the same cycle, seven m2 and five A1. No scale in this list has more than twelve notes.

6. The scales that are not explained

- **"Whole-tone", the two diminished scales, "Augmented (hexatonic)" and the "Tritone scale"** repeat inside the octave of 12-edo. Six M2 steps do not sum to P8 on the stack of fifths (they sum to twelve fifths less six octaves), so the "Whole-tone" scale closes only when one step is written as a d3. These scales are properties of 12-edo's equal division, not of the diatonic scale.
- **These collections are sets of dominant seventh chords.** In 12-edo the scale "Diminished, half step first" is exactly the four dominant seventh chords whose roots lie an m3 apart, and the "Tritone

scale" is exactly one dominant seventh chord and its substitute dominant (the jazz "tritone substitution", whose root lies six 12-edo steps away). The script `jazz_diminished_by_chord.py`, beside this file, measures which chords the diminished and "whole-tone" passages of jazz solos sit over; its output is `jazz_diminished_by_chord_output.txt`.

- **Hungarian major** is, in 12-edo, seven of the eight notes of the scale "Diminished, half step first", so it belongs with that scale.
- **Enigmatic** is a composed puzzle scale, which Verdi set in his *Ave Maria*; no rule explains it.

7. Limits

- **The table maps note sets, not practice.** *Two Scales That Are Not Scales* measures that harmonic minor does not operate as a scale in the vocal repertories that paper samples (c. 1420–1649). This table shows only that the jazz scales' notes are the notes the rules produce. Whether jazz lines place those notes where the rules place them is a corpus measurement that this table does not make. The script `symmetric_and_a2_measures.py`, beside this file, measures the scales of section 6 and the A2 as a melodic step in the corpora on this machine; its output is `symmetric_and_a2_measures_output.txt`. The script `makam_a2_and_jazz_measures.py` measures the same things in the jazz solos of the Weimar Jazz Database and in the Turkish makam scores of SymbTr; its output is `makam_a2_and_jazz_measures_output.txt`.
- **Class 6 is another mode of a minor mode with rules applied.** The rules are stated at the positions of a final cadence, and say nothing about taking another note of the result as the tonic. Jazz teaching itself states a rotation as one set heard over another chord: it commonly gives the altered scale as the melodic minor scale an m2 above the chord's root.
- **The explanations depend on spelling.** The altered scale is a melodic minor mode only when its fourth degree is spelled ♭4. Spelled by chord tones (♭9, ♯9, 3, ♯11, ♭13, ♭7) it spans fourteen fifths and is not explained; the two spellings are the same notes only in 12-edo. *Jazz in MeloPresto*, in its section 10, lists the spelling of the ♯9 as open.
- **The list is not closed.** The diatonic modes, the melodic minor modes, the scale "Diminished, half step first" and the altered scale follow the note names in Wikipedia's article *Jazz scale*; seven degree formulas follow Wikipedia's *List of musical scales and modes*. The other formulas are the standard ones of jazz scale teaching (Aebersold's *Scale Syllabus*, Levine's *The Jazz Theory Book*) and were not checked against those books for this table. Any further scale can be added to `SCALES` in the script.